A New Look at the Semantics and Pragmatics of Numerically Quantified Noun Phrases

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Abstract

This paper presents some arguments against a unilateral account of numerically quantified noun phrases and for a bilateral account of such expressions. It is proposed that where numerically quantified noun phrases give rise to *at least* readings, this is the result of one of two forms of pragmatic reasoning. To that end, the paper develops an independently motivated account of specificity and existential closure involving diagonalisation.

1   Introduction

In this paper the semantic and pragmatic properties of numerically quantified noun phrases (NQNPs) are considered. In particular, we will consider the status of implications like those in (1b) which are normally available from assertions of the example in (1a):

(1) a. Two of the students did well on the test.
     b. No more than two of the students did well on the test.

According to one commonly discussed view, NQNPs have a lower-bounding linguistic meaning giving rise to an *at least* interpretation.[1] That is, an interpretation of (1a) based on an understanding of its linguistic meaning alone could be characterised according to the equivalence in (2) - where F denotes the set of sets of students and G denotes the set of sets of individuals who did well on the test:

(2) Two of the students did well on the test. $\Leftrightarrow \exists X[|X| = 2 \wedge F(X) \wedge G(X)]$

According to this view, where an upper-bounding implication as in (1b) arises it is made available through conversational implicature, based on a (neo-)Gricean Quantity maxim. While the same kind of account is widely accepted for quantified noun phrases such as 'some students' and 'many students', this view of NQNPs has never really commanded a firm consensus because their interpretation does not clearly conform to the analysis. In fact, there is a great deal of evidence which suggests NQNPs would be better paraphrased using 'exactly'. This evidence will be reviewed below. However, while the evidence is

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[1] In what follows, single quotes (e.g. 'two') are used to make reference to expressions of English. Italics (e.g. 'an *at least two* reading') are used to make reference to particular interpretations.

strong, no alternative to the standard view of NQNPs has been widely accepted. The challenge to any account of NQNPs as encoding an *exactly* interpretation stems from the fact that noun phrases containing numeral expressions still often give rise to *at least* readings. As such, any *exactly* account would be bound to explain the latter cases in terms of pragmatics. Until now, no such account has been forthcoming. In this paper, I will defend the *exactly* analysis of NQNPs and provide an account of the *at least* cases in terms of independently motivated pragmatic inferences concerning specificity and existential closure. We will also see that there is little independent motivation for arguing that NQNPs are simply ambiguous between the unilateral, *at least* interpretation and the bilateral, *exactly* one. As such, the bilateral-only account ought to be favoured.

The proposals for NQNPs in this paper build on a set of ideas concerning specific uses of quantified noun phrases. In these cases, the speaker exploits a pervasive form of pragmatic reasoning best known in semantics as diagonalisation. It will be argued that the same pragmatic account is independently required for exceptional scope data (which includes exceptional-scope, *at least* readings of NQNPs) and for cross-sentential anaphora (including anaphora dependent on specific NQNPs with *at least* readings).

In the following sections, the standard, unilateral view of how sentences containing NQNPs are interpreted will be presented first. Subsequently, we will see that while a parallel unilateral view of NPs involving other quantity expressions would seem to be on the right track, the unilateral account of NQNPs has quite serious problems. It will be argued that, unlike other QNPs, NQNPs behave as though their linguistic meaning encodes the bilateral, *exactly* interpretation. We will then explore how, assuming a bilateral linguistic meaning, various semantic and pragmatic factors determine the different interpretations of utterances containing these expressions.

2   The Unilateral View

There have been many proposals that the implication (1b) is an implicature premised on some Quantity maxim (Gazdar 1979, Horn 1989, Levinson 1983, van Rooy & Shulz 2006). Although these accounts differ, they are at one in ensuring that the implicature does not normally arise in downward entailing (DE) contexts; for example, that the implication in (1b) does not fall within the scope of the conditional in (3):

(3)   If two of Mary's students did well on the test, then Mary won't be fired.

This is reasonable when we look at other triggers for quantity implicatures. For instance, (4a) might often imply that no one other than John did well on the test. This exhaustive implication does not attach to the antecedent of the conditional in (5a). Mutatis mutandis, (4b) tends to imply that not all of the students did well, while this implication disappears from the antecedent in (5b):

(4)   a.   John$_F$ did well on the test.
       b.   Some of the students did well on the test.

(5)   a.   If John$_F$ did well on the test, then Mary won't be fired.
       b.   If some of the students did well on the test, then Mary won't be fired.

This makes sense from a Gricean perspective since implicatures based on the Quantity maxims turn on there being a more informative alternative *utterance*. In the case of 'Some of the Fs have G' this alternative is 'All of the Fs have G'. Where there is embedding in a conditional, the alternative, 'if All of the Fs have G then P' is no longer more informative and so there is no ground for this kind of implicature. We can characterise a unilateral view as follows:

Unilateral View: *[two N]* has unilateral lexical meaning (as suggested in (2)) and common upper-bounding implications arise as Quantity-based conversational implicatures.

3   Problems for the Unilateral View

A great many observations have been made which suggest that numerals are different to other scalar items. It seems that NQNPs do not behave as if they have unilateral meaning[2]. Here I will add some observations of my own (in section 3.1) in a survey of some of the points against the unilateral view to be found in Carston (1988, 1998), Horn (1992, 1996), Geurts (2005) and elsewhere.

3.1   Downward Entailing Contexts

As suggested in relation to (4) and (5) above, conversational implicatures based on a Gricean Quantity maxim ought not to arise where the triggering sentence appears in

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[2] A note on terminology: The terms 'linguistic meaning of α' or 'meaning of α' will refer to the semantic interpretation(s) of α as determined by the semantic rules of the grammar as applied to the expression (in the imagined context under discussion). For instance, according to the unilateral account, the meaning of the NQNP *[two students]* can be given as $\lambda P.\exists X[|X| = 2 \land students(X) \land P(X)]$. The term 'reading of α' will make reference to the interpretation of α when uttered in the imagined context under discussion. We will often use paraphrases to characterise readings and make reference to paraphrases in the short-hand terms noted above, viz 'an *at least* reading' or 'an *exactly* reading'. On the unilateral view, the typical reading of [[two students][failed]] could be paraphrased as $|\bigcup\{X: students(X) \land failed(X)\}| = 2$ (an *exactly* reading) due to the fact that the reading in question combines the interpretation as determined by the meaning of the sentence plus an upper-bounding implicature to the effect that no more than two students failed.

downward entailing contexts (at least not without special intonation - see below). A brief consideration of (3) above suggests that the unilateral view of NQNPs would be confirmed in these kinds of contexts as well. However, a closer inspection of these cases reveals some more problematic data. Consider the examples in (6a-d):

(6)   a.   Everyone who has two children receives tax benefits.
                [Implies everyone who has three or more children receives benefits]
       b.   Everyone who has two children received tax benefits.
                [Implies everyone who has three or more children receives benefits - but not so strongly]
       c.   No one who has two children received tax benefits.
                [Does not imply no one who has three or more children received benefits]
       d.   No one who has two children receives tax benefits.
                [May imply that *everyone* with three, four, etc. children receives benefits]

If the unilateral view is right, these should all have implications along the lines of (6a). But they do not. In particular, (6d) seems to imply that people with more than two children *receive* benefits. Even the suggested implication of (6a) seems to turn on an implied or presupposed generalisation concerning how tax benefit rules are devised.[3] Consider (7) by contrast where no such covering generalisation would normally be presupposed:

(7)   Everyone who has two children is happy.

It seems that on its most accessible reading, what (7) says would be true if John is miserable but has five children. Note that, by contrast with (7), (8) seems to quantify over people who have children and pets in addition to those who have only children and those who have only pets - as we would expect with the normal unilateral, inclusive interpretation of disjunction:

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[3] Throughout this paper the terms 'presupposition' and 'presuppose' express thoroughly pragmatic concepts and are not meant in any way to be related to linguistic presuppositions. A pragmatic presupposition is an assumption that the speaker assumes is common ground among conversational participants, where 'common ground' is to be characterised in terms of structures akin to that of common knowledge or common belief (see Stalnaker 2002). (The only exception to this usage comes later in the paper in the context of mentioning van der Sandt's and Geurts' accounts - where their claim is that presupposition is linguistically driven.)

(8)   Everyone who has children or pets is happy.

Indeed, it is easy to show how NQNPs should behave if the unilateral view is right by considering disjunction in contexts similar to those in (6). In each of (9a-d) the domain of quantification seems to include people who have children *and* pets. I.e. the negative quantifier in the (c) and (d) cases makes no difference:

(9)   a.   Everyone who has children or pets receives tax benefit.
        b.   Everyone who has children or pets received tax benefit.
        c.   No one who has children or pets received tax benefit.
        d.   No one who has children or pets receives tax benefit.

These data should make us suspicious. If the unilateralist view is right, then we should understand all of these examples with downward entailing contexts in the same way as (9) - where 'two Fs G' has its unilateral interpretation. In fact, what we find for NQNPs is that only in those cases where we can make some kind of assumption about a covering generalisation do we get an understanding that is consistent with the unilateralist view. This is brought home by considering (10a) in two contexts (10b) and (10c). In Context A, there is a suggestion that people who have more than three children are also unhappy, whereas Context B pushes the invited implication the other way - that people with more than three children are happy:

(10)   a.   No one who has three children is happy.
        b.   Context A: People in the society under discussion tend to be more stressed the more children they have.
        c.   Context B: The society under discussion is poor and more children means more prosperity.

Indeed, contrary to the unilateralist view, it seems that this data could be better explained if we assumed that the linguistic meaning of NQNPs is bilateral. That is, if instead of using the equivalence in (2) to gloss the literal interpretation, we use (11):

(11)   Two of the students did well on the test. $\Leftrightarrow |\bigcup\{X: F(X) \wedge G(X)\}| = 2$

In the case of (6a), the idea is that the example evokes a context where one is being informed about the tax benefit rules. To get the enriched...*or more* reading, we make a background assumption to the effect that tax benefit rules are normally couched in terms of a lower limit: Anyone who has a number of children above the lower limit also qualifies for the break. Given that the speaker would know that this is relevant and that the audience would assume that the speaker knows about such assumptions and that the speaker has done nothing to forestall such assumptions being made, it will become common ground that the speaker has made this assumption manifest. Hence the

assumption becomes what might be termed a background implicature and can be presupposed.[4] Given the now presupposed background assumption and what has been asserted, the *or more* implication follows.

In addition to the...*or more* implication, there is a further inference to be made that no one with just one child receives benefit. This inference is derived as a quantity implicature on the bilateral account: Given that, in context, a key implication of the assertion is clearly intended to be the proposition that two or more children is sufficient for a tax break, the question naturally arises whether one child is sufficient. Given the relevance of this question to the topic and given the speaker's apparent knowledge of the tax laws and willingness to communicate them and the fact that the speaker could have used 'one' instead of 'two', one can infer that having one child is insufficient (by standard quantity-implicature reasoning).

Note that this quantity implicature is different from that which would be derived as a neo-Gricean 'reverse-scale' scalar implicature, given the unilateral interpretation of NQNPs. In that case, the alternative, 'Everyone with one child receives tax benefit' is a more informative alternative and thus, only its negation would be implicated (see Levinson 2000 for an account of these 'reverse scale' scalar implicatures).

In addition to not correctly deriving the lower-bounding quantity implicature for (6a), the unilateral view does not have anything at all to say about (6d) except that it should pattern with (6a) and so it should entail that no people with three, four, etc. children get tax benefits. In addition, it should scalar implicate that some people with one child get tax benefit . But (6d) does not pattern with (6a) at all. (6d) implies that no one with one child gets tax benefit and it can, in the right context, imply that everyone with three or more children gets tax benefit. Neither of these implications seem explicable on the unilateral view. However, on the bi-lateral view, the implications are derived in the same way as those for (6a). According to the bilateral account, the same background assumptions about the structure of tax laws get to be presupposed (as above). Given this context, the assertion with the bilateral meaning for 'two children' implies that no-one who has one child gets the tax break. Given this same context, one can notice that the speaker's uttering the sentence, 'No one with three children receives tax benefits' would have been more informative since it would imply both that no-one with two children and no-one with one child receives benefits. Hence, by familiar quantity implicature reasoning, one can infer that the speaker did not utter this because it is not true and that having three children is sufficient for tax breaks.

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[4] See Grice (1975) and Sperber & Wilson (1986) where the ideas are introduced. See also Simons (2006) for some recent discussion of the connection between background implicatures and presupposition.

3.2   Implicature 'intrusion' under denial (cf Horn 1996)

Given that in many of these downward entailing cases our intuitions would be explained if we assume that NQNPs are interpreted bilaterally, there is another problem for the unilateral view: normally, bilateral interpretations of scalar terms in DE contexts involve a marked operation (needing focus or special contexts). Compare the exchange in (12a,b) and (12a,b') - note: CAPS indicate special rising, contrastive intonation:

(12)   a.   Mary: John got some$_F$ of the questions right.
         b.   Bill: He didn't get SOME$_F$ of the questions right.
         b'.   Bill: He DIDn't$_F$ get some of the questions right.

Assume that Mary utters (12a) with focus on 'some' and that she and Bill had been wondering how John would do on a particular test. In such a context, the implication that John didn't answer all of the questions would be available. The natural construal of (12b) is that Bill is suggesting John didn't answer some and not all of the questions but in fact he answered all of the questions. No such construal of (12b') is available. This would just be understood as a denial of the existential claim Mary has made.[5] Now compare (12) with the parallel case in (13):

(13)   a.   Mary: John got four$_F$ of the questions right.
         b.   Bill: He didn't get FOUR$_F$ (of the questions) right.
         b'.   Bill: He DIDn't$_F$ get four (of the questions) right.

Here, Mary would naturally be understood to mean that John got just four questions right. But this time Bill's response in either (13b) or (13b') could be understood to be denying the proposition that John got just four questions right. Moreover, in (13b), it does not seem right to say that Bill is simply denying the upper-bounding inference that he got no more than four questions right; rather the contrastive stress would suggest simply that he got some other number (maybe five, maybe three) right. The intuitions are even clearer for these examples if Bill responds more naturally with the elided, 'No he didn't'.

   The idea that (13b') might further imply that John didn't get five, six or more questions right would only arise in a context where it is assumed that the test was quite difficult, that John wasn't expected to get as many as four questions right. However, if there were one hundred questions and four is below expectations, then (13b) no longer has this implication.

   In summary, there is unexpected behaviour of NQNPs in DE contexts. In contrast to 'some' and 'or', it does not look like the favoured reading in DE contexts is the unilateral

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[5] Why focus is required on the quantity expression in order to deny the upper-bounding implication is not a question which is settled. One popular account is that negation in this case is metalinguistic. See Horn (1989) for an early discussion.

reading. It looks like the *at least* reading for NQNPs in DE contexts turns on special implicated generalisations. This weakens the case for the unilateral view since the *at least* reading should be the favoured or default reading in DE contexts. Where there is a bilateral interpretation of NQNPs in DE contexts, unilateralists would have to concede that some form of implicature intrusion is going on. However this form of intrusion requires special intonation in the case of other quantity expressions but not numerals.

3.3   Questions (see Horn 1992)

Consider the possible answers to the question in (14a). The markedness of the answer to the *at least* construal of the question is in contrast to the opposite pattern of markedness in the 'some' and disjunction cases.

(14)   a.   Do you have three children?
          b.   No. I have two.
          c.   No. I have four.
          d.   ?Yes. (In fact) I have four.

(15)   a.   Do some of your friends have children?
          b.   ?No. All of them do.
          c.   Yes. (In fact) all of them do.

(16)   a.   Did John or Mary pass the exam?
          b.   ?No. They both did.
          c.   Yes. (In fact) they both did.

On a Gricean view of scalar implications, the oddness in (15) and (16) makes sense, since in the context of questions, the quantity maxims do not apply (questions in fact set the level of informativeness required). So, the pattern in (14) is unexpected if NQNPs have a unilateral meaning.

4   Alternatives to the Unilateral View of NQNPs

As it becomes more widely accepted that the unilateralist position for NQNPs is not well supported, the question arises as to what alternatives are potentially available and whether any of these have been put forward. Although Horn (1992) provides much evidence for the fact that NQNPs are not the same as other scalar terms, no positive account is offered.

Carston (1998) entertains two analyses of NQNPs. One is the bilateral account being defended here. The other is an underspecification account. According to the latter, whether a NQNP is understood as *at least n*, *exactly n* or *at most n* could be left

unspecified by the meaning of 'n Fs' - just as the relation between John and the book in 'John's book' is unspecified by the meaning of the possessive construction. Although Carston expresses a mild preference for the underspecification account, she leaves the matter open.

Regarding Carston's favoured alternative, some of the data discussed above suggest that the underspecification account may not be correct. For example, consider the asymmetry in downward entailing contexts. Given the underspecification proposal it is open whether NQNPs have a unilateral or bilateral interpretation and only extra-linguistic, pragmatic principles are the determining factor. Now it could be argued (see Winter 2001) that where there are two logically related candidate interpretations of an expression, the interpretation which is favoured, *ceteris paribus,* is that which makes the overall proposition expressed logically stronger. In the DE case, the stronger interpretation would be the unilateral interpretation. But intuition suggests that in DE contexts the bilateral interpretation is favoured. That is, unless there is a supporting generalisation, one reads NQNPs in DE contexts bilaterally. If, on the other hand it were not correct to assume that stronger interpretations are favoured, the underspecification account ought to predict no preference, contrary to intuition. Either way, these intuitions are problematic for the underspecification account.[6]

Geurts (2006) favours what is referred to as a polysemy account of 'two' whereby each of three meanings is listed in the lexicon: a predicational meaning, a bi-lateral quantificational meaning and a unilateral, existential quantificational meaning. Geurts observes that these meanings are related to each other via type coercions that relate predicational and quantificational interpretations of descriptions generally (see Partee 1986). Starting with the bilateral quantificational meaning for 'two students' represented in (17a)[7], we can derive the predicational meaning (17b) via the *BE* operator $(\lambda F_{((et)t)}\lambda x[F(\lambda u.u = x)])$ and in turn the unilateral quantifier meaning (17c) via *A* $(\lambda P.\lambda Q[\exists x[P(x) \wedge Q(x)]])$:

(17) a. $\lambda P.\exists!x[\#x = 2 \wedge students'(x) \wedge P(x)]$
      b. $\lambda x[\#x = 2 \wedge students'(x)]$

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[6] Carston's proposal may seem to have an advantage over the current one since it also covers the *at most* reading of NQNPs that arises in modal and related contexts - eg (i):

(i)   I can fit four people in my car

However, it should be apparent that the *at most* reading can be derived as a quantity implicature given a bilateral meaning for NQNPs. Cf the discussion of (6a,d) above.

[7] Here quantification is over group individuals; # is a function from such individuals to the cardinality of the group and $\exists!x[...$ reads, 'there is a unique x...'

c. $\lambda P.\exists x[\#x = 2 \wedge students'(x) \wedge P(x)]$

It should be noted however that while (17a) is related to (17b) and (17b) to (17c) via the kind of coercion that can be found relating other descriptive expressions, (such as Geurts' example, 'students') there is no way back from the predicational (17b) to the bilateral meaning (17a) via one of Partee's 'regular' coercions. In fact, there is another of Partee's type shifts that is available to operate on (17b) and that is THE. This operator yields the meaning of 'the two students'. That (17a) does not fit the regular Partee pattern squares with Geurts' conjecture that the basic, underlying meaning of 'two' is the bilateral quantificational determiner meaning. One assumes that the other two understandings of 'two students' mentioned arise because it is normal in English (and other languages) that such shifts in meaning relate one sense of an expression to another. To illustrate this point, Geurts cites the case of the count/mass shift that can relate senses of 'beer' and 'apple'. Each nominal has its own basic sense - respectively mass and count. Yet via what might be called regular coercions one can derive the, respectively, count and mass meanings of the expressions. To sum up, even though the basic meaning of 'two students' is the bilateral, the expression is simply ambiguous between the three meanings described above. This ambiguity arises due to the presence of regular coercions in English.

While this account looks neat, the argument involving regular coercions to motivate the polysemy account has an interesting and problematic corollary. Let us take another look at the structure of that argument: We grant that 'two' is a non-monotone determiner but we also have to recognise that 'two' can appear as a modifier in predicate constructions as in, 'those are two cows'. We recognise that, generally, modified nominals, like 'brown cows', which can appear in predicate position can appear in argument position and take on existential force, as in 'Brown cows were standing in the field'. By analogy, we can reason that 'two cows' in argument position can undergo the same existential closure. Thus 'Two cows were standing in the field' ought to have an *at least* meaning.

While this line of reasoning may seem appealing, bear in mind that modified numerals can also appear as predicate modifiers:

(18)   a.   Those are exactly 50 books on descriptions.
       b.   We need to gather 50 cows. Those are at most 20. So we need to find at least 30 more cows.

Similarly, in as far as bare numerals can function as predicates in English, so can modified numerals:

(19)   a.   We are three.
       b.   We are exactly ten.
       c.   If the guests arrive, they will be at most three.

Given that predicates can be modified by 'exactly n' and 'at most n' and by the reasoning applied for bare numerals, there ought to be a reading of 'exactly two students' or 'at most three cows' which is derived by a form of existential closure on such a predicate. But, presumably, as a predicate, 'exactly two' applies to collections of exactly two individuals while 'at most two' applies to collections containing two, one or zero individuals. This means that there ought to be a grammatically derived meaning of, 'Exactly two students asked a question' according to whose truth conditions the sentence is true where three students asked a question. There also ought to be a meaning of, 'At most two students asked a question' which is false if no students asked a question and true if three students asked a question. All of this is to suggest that the line of reasoning currently under consideration is problematic. What relationship there is between predicates modified by numerals in various ways and the homophonous argument noun phrases is not as straightforward as it may first appear.

Another conclusion to draw from these considerations is that Partee's coercions really cannot be considered sytematic, grammatically licensed shifts in meaning that simply apply across the board. Of course, Geurts' proposal does not presume this. In fact Geurts is explicit that the various senses of 'two' are listed separately in the lexicon. We can indeed separate out the proposal about the polysemy of 'two' from its motivation. However, in doing so we remove any independent motivation for the idea that 'two cows' in argument position is ambiguous between a bilateral and an existential unilateral meaning.

We will return to the differences between bare NQNPs and modified versions like 'exactly two students' once we first flesh out the picture of how one could naturally derive the attested *at least* readings of NQNPs given a bilateral meaning. We begin with a survey of some further examples.

5.   Genuine Unilateral Interpretations?

Thus far, we have encountered a number of examples where NQNPs might be glossed using the forms 'at least...' or '...or more' where it seems this gloss arises as a form of implicature given certain background assumptions. Let us reconsider (6a) which could be glossed, 'Everyone who has two or more children...':

(6)   a.   Everyone who has two children receives tax benefits.

It was suggested above that this *at least* reading could just as well arise given an *exactly* meaning of the NQNP via a background implication/presupposition about the normal structure of tax laws. In addition, the unilateral view has nothing to say about the...*or less*

reading of (6d) while it seems clear that this reading could be derived in the same way as the *...or more* reading of (6a) given the bilateral account.

(6)   d.   No-one who has two children receives tax benefits.

Some more evidence for this account of these examples can be found in a study of children's understanding of rules such as the following, reported in Musolino (2003):

(20)   If you get two hoops over the stick, you get a prize.

Asked if they should reward a puppet who got three hoops over the stick, younger children in Musolino's study tended to hold back the reward, while they were happy to reward puppets who got only two hoops over. Children of this same age group generally treat NQNPs bilaterally in truth-judgement tasks involving assertions such as 'Three horses jumped the fence'. At the same time, these same children very strongly favour a non-bilateral interpretation of 'some horses jumped the fence' - assenting to the statement when in fact all the horses jumped (Musolino *ibid*, Papafragou and Musolino 2003). The data suggests that, for whatever reason, children of four or five years find it difficult to derive implicatures of various sorts. On the other hand, it has been independently shown that children universally derive the unilateral reading of disjunction and 'some' in negative contexts such as the antecedent of conditionals (see Gualmini & Crain 2002). So if NQNPs had unilateral meanings, children ought not have trouble with Musolino's task involving (20). But children do and it seems that while they start off with a unilateral meaning for disjunction and 'some', they start off with the bilateral interpretation for NQNPs. The reason for their odd responses for (20) would then be explained as a failure to derive the *...or more* implication via the background implicature about the structure of rules of such games.

It is interesting that Musolino was able to get an *at least* response out of children of the same age group in a different kind of task. In this second study, one puppet requires two cookies for a certain purpose and seeks to buy them from a second puppet, Goofy. In the critical trials, Goofy has four cookies to sell and children in the age group tended to assent to the final question in the following discourse:

(21)   Let's see if Goofy can help the Troll. The Troll needs two cookies. Does Goofy have two cookies?

Musolino notes that this new design is motivated by an example from Kadmon (2001) of the following type:

(22)   There are four chairs in the seminar room.

Normally, an assertion of (22) would be understood to mean that there are exactly four chairs in the seminar room. However, in the kind of context Kadmon discusses, a person is looking for four chairs (needed for a meeting for instance) and tells you so. You utter (22) when all you know is that there are many more than four indistinguishable chairs in the seminar room. In this context, it seems clear that, for all we learn from the speaker, there could be more than four chairs in the seminar room. However, we in fact do not really learn from the speaker anything about any number greater than four. This kind of example is clearly different from (6a) and (20). In fact it is different to the extent that we would be reluctant to gloss what the speaker means using 'or more'. That is, we would be reluctant to use 'There are four or more chairs in the seminar room' as a gloss of what the speaker means. Rather, what the speaker means is something like, 'There are four chairs *for you* in the seminar room' or 'There are four chairs *for your purpose* in the seminar room'. In the other example, when we include the implicature the speaker means that if you have two *or more* children then you qualify.

What we can say about the difference between the Kadmon example, (22), and examples such as (6a) is that the former involves a form of domain restriction while the latter involve a form of conversational implicature. This would explain young children's differential performance on tasks based on the different kinds of example: while children do not have difficulty with domain restriction (they must encounter quantificational phrases that require domain restriction all the time), they do have difficulty with genuine conversational implicature of the kind found in (6a). Having said that, it is legitimate to wonder what it would mean to say that there are four cookies for the puppet in the presence of six cookies that are equally available for sale. Which four cookies are at issue? The answer for the children in Musolino's study may be that they partition the cookies in their mind yielding a set of four cookies for the puppet. In fact it ought not suprise us if a child playing this game insists on a certain four cookies when the puppet goes to collect his purchase. In general, we can see different collections of four cookies as being the ones that the puppet receives for its purchase and we can accordingly identify different potential goal states for the puppet - one for each possible set. But given that we do not know which four cookies the puppet will receive or even whether the puppet in fact does receive four cookies, how is it that we can talk about the four cookies for the puppet? The answer to this question that will be expounded at length below involves a form of pragmatic reasoning known variously as pragmatic assent, reflexivisation or diagonalisation (see Perry 2001, Stalnaker 1978). Effectively, the idea is that we take for granted that there were four specific cookies already identified in making the assertion and, without knowing which four have been identified, see what would follow from the assertion. In this case what follows from the assertion is that the puppet can realise its goals through a cookie purchase.

A number of 'at least' examples discussed in the literature seem to pattern the same way. Consider the following example from Carston (1998):

(23)   A: If you have two children you qualify for child benefit.
          B: I have two children. (In fact I have three.)

In a sense, we could gloss B's response as, 'I have two children for the purposes of fulfilling the child benefit rules' and it is easy to see how, if NQNPs have a bilateral meaning, this reading could be derived along the lines of the Kadmon example.

The strategy for the bilateral account then is to argue that *at least* readings can arise either through a kind of background implicature or via this process of reflexivisation. An important class of cases to be considered involve modal contexts - (24a) is from Carston (1998), (25a) is from Geurts (2006):

(24)   a.   I predict twenty people will be there tonight.
          b.   I bet that five women finish in the top one hundred in this year's marathon.

(25)   a.   You must take two cards.
          b.   To qualify for this course, you must have two A grades.

Probably the most accessible understanding of (24b) and (25b) is the *at least* one while (24a) and (25a) are more clearly ambiguous in that we can imagine contexts where they give rise to an *exact* reading just as easily as contexts where the *at least* reading is intended. Examples like (25b) pose a challenge to any bilateral account of NQNPs since here we seem to be able to obtain an *at least* reading where the NQNP is part of a necessary condition. Strictly speaking, having exactly two A grades as a necessary condition for qualification would seem to preclude having three. By contrast, having two children as a sufficient condition for tax breaks (as per (6a)) does not preclude having three children and qualifying. Whereas the addition of an implicature to the meaning of (6a) narrows down the overall interpretation in the appropriate way, it does not seem that a similar move would be possible in the case of (25b). To compound the challenge for the bilateral account, we can observe that the main quantity implicature for (25b) is that one does not need more than two A grades to qualify. This is the implicature one would derive if (25b) were to be equivalent to $\Box \exists x[\#x = 2 \wedge \text{A-grades}(x) \wedge \text{you\_have}(x)]$, since the alternative, 'You must have three A-grades' would entail what is asserted and thus its negation would be implicated.

Before getting too enthusiastic on behalf of the unilateral approach given this example, we should note first that, like (6a), the reading of (25b) needs the appropriate kind of background assumptions about the nature of the underlying rules. Secondly, the *exactly* readings of these examples, such as where (25a) is understood to mean you must take just two cards, are not derivable via standard Gricean reasoning given a unilateral meaning. At best, the examples could be seen as motivation for the ambiguity approach.

It will be argued below that these examples can be seen as exploiting the same form of reasoning as sketched above for the Kadmon examples. Moreover, in cases such as these, the form of reasoning effectively constitutes a kind of pragmatically-derived existential closure. Motivation will be provided in favour of this pragmatic route over the grammatically encoded route. The motivation will come from exceptional scope facts and facts about anaphoric relations - topics to which we now turn.

6.   Specificity and Existential Closure

6.1   Preliminaries

Suppose that, in talking about what happened in a popular park on a Sunday afternoon, the speaker utters (26):

(26)   Two men were walking in the park.

You know that it is unlikely that the speaker is claiming that just two men were walking in the park; rather, the natural understanding of this kind of speech act is as a piece of discourse introducing two men. Which two? In some cases, but probably not so many, it may not matter which two and, as in Kadmon's chair example, you can just assume that the two men are given and see what follows from that. In other cases, you might assume that the speaker stands in some kind of epistemic relation with two specific men and it is these that the discourse is about. These would be the two men the speaker has 'in mind' in producing this token of the noun phrase. The latter kind of specific reading comes to the fore where there are continuations involving anaphora:

(27)   Two men were walking in the park. They were whistling.

While this way of looking at things suggests that 'two men' is bilateral and specific, someone who is tempted by either the unilateral account or the ambiguity account could claim that (26) and (27) both involve the existential, unilatateral meaning of 'two men', as per Geurts' proposal in (17c) above. The proposal would be that in (27), the quantifier is dynamic, binding the plural pronoun in the second discourse segment (see Groenendijk & Stokhof 1991, Kamp & Reyle 1993, Chierchia 1995 for some ideas of how this might work). However, this type of example has been discussed at length in the literature on dynamics and anaphora where it has been observed that a simple dynamic binding relation between 'Two men' and 'they' cannot capture the fact that the anaphoric relation is mediated by the specificity of the antecedent and the definiteness of the anaphor (see Stalnaker 1998, van Rooy 2001, Breheny 2004). For example, while a natural continuation of (27) by an interlocuter might involve so-called pronominal contradiction,

as in (28), this cannot be accounted for in any kind of dynamic binding approach:[8]

(28)   They weren't men, they were robots, and they weren't walking, they were rolling on little wheels.

So here we have a case where one might gloss a sentence containing an NQNP with 'at least' but where there is good reason to think that the NQNP itself is understood bilaterally but specifically.

In this section we will consider those *at least* examples discussed above that cannot be handled via the background implicature account to see if, like Kadmon's example, they may be explicable as specific noun phrases that are interpreted by a pragmatic reflexivisation process. In order to make the discussion more concrete, we first need to make some assumptions about what it means for 'two men' to have a bilateral meaning and for such a noun phrase to be understood specifically. This we will do now and in addition, we will go over the two-dimensional framework for representing the pragmatic reasoning behind (27), as introduced in Stalnaker (1978) and elsewhere.

We assume that, as an argument NQNP, 'two Fs' is assigned a meaning by the semantic rules of English along the following lines:[9]

(29) $\lambda G. \left| \bigcup \{Z: F(Z) \wedge G(Z)\} \right| = 2$

Turning now to the question of specific noun phrases, there have been a number of proposals in the literature about their formal treatment. Most notably, specific indefinites have been analysed semantically using Choice Functions (see Reinhart 1997, Kratzer 1998, Winter 2002 and Schlenker 2006 among others). Among these, we can distinguish between Kratzer's account and the others. For Kratzer, specifics introduce a free choice-function variable while for other accounts a mechanism of existential closure is employed in deriving the semantic interpretations of sentences containing specifics. We will discuss existential closure accounts as they would apply to NQNPs a little later. Among non-choice-function accounts, that which is found in Schwarzschild (2002) is in interesting ways similar to Kratzer's. Schwarzschild (2002) advocates the 'singleton indefinite'

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[8] This is not the only problem facing the dynamic treatment of such examples and the reader should consult Stalnaker (1998), van Rooy (2001) and Breheny (2004) for more reasons to favour the specific analysis of the indefinites in these cases.

[9] This way of representing the bilateral meaning of NQNPs as Generalised Quantifiers is the most 'neutral' in the sense that it allows for a reading of 'Three men lifted the piano' which is true if John singlehandedly lifted the piano and Bill and Sam also did. We can also assume, following Schwarzschild (1996) and others that distributivity, when it occurs, is a property of the predicate, here G.

approach. Effectively, Schwarzschild's proposal is that indefinites are quantified noun phrases but, when they are specific, they are understood to have a contextually restricted domain of quantification containing just one individual. So, 'a man' is understood specifically where the domain of quantification is taken to have been reduced to a unit set. In the case of 'two men' the idea would be that the domain of quantification is a singleton collection, but, given the meaning in (29) above, this singleton collection would be of two men. Although Schwarzschild's proposal is formally distinct from that of Kratzer (1998) the two have an important property in common when contrasted with other choice function accounts. This is that they presume that the speaker can express a proposition in context knowing that the audience cannot grasp what that proposition is through ignorance of the value of some kind of contextual parameter.

For Schwarzschild, all one knows about the contextually restricted domain of quantification for the indefinite is that its extension consists of just one member. But Schwarzschild acknowledges (following Stanley & Szabo 2000) that quantified noun phrases require that one supply the intension of the domain for a full interpretation. For example, where the speaker uses 'every student' intending to quantify over students in Bill's class, then what context supplies is not simply the set of individuals in Bill's class but a function from worlds to the set of whoever is in Bill's class. Thus, on Schwarzschild's account, the audience often does not know what the full, contextually determined interpretation of 'some student' is when it is used specifically.

In a similar vein, on Kratzer's account, when 'some student' is understood specifically, the noun phrase is analysed so that a choice function variable is sister to the nominal predicate - understood as providing the argument for the choice function. Thus, when a speaker uses 'some student' specifically, in order to grasp what proposition is expressed, one has to know which function serves as the value of the variable and hence one should know which individual the indefinite denotes. But often one does not, as we will see shortly.

Normally, it is expected of the speaker that the audience can grasp the full set of truth conditions for the sentence s/he utters. For example, if 'John' is used, then the speaker should ensure that, in context, the audience can figure out which John is referred to. Similarly the audience should be able to infer the referents of pronominals and demonstratives and the domains of quantified noun phrases. These expectations are summed up in Grice's Manner maxims enjoining clarity and the avoidance of ambiguity. In two-dimensional terms, Stalnaker (1978) proposes a principle of conversation which also captures this expectation: this is the principle that the speaker express the same proposition in each context alternative.[10] In following Schwarzschild's or Kratzer's

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[10] In fact this is one of three principles proposed in that paper. Although Stalnaker suggests his principles are all broadly Gricean in spirit, his motivation for the principle in

suggested analysis here we should note that their proposals imply that speakers openly violate these expectations in the case of specifics. This raises the question why such violations are condoned in this kind of case.

We can get an answer to this question by noting firstly that in *openly* violating the expectation, the speaker is at least at some level being clear (about not being clear). Second, as suggested in the discussion of Kadmon's example, it is not always necessary to determine exactly what proposition is expressed. Sometimes, one can take the truth of that proposition for granted (without knowing exactly what it is) and see what follows from that. If what does follow for the hearer is sufficient for the conversational purpose at hand then, in some sense, the speaker has respected the Gricean principles at the level of what is implied (or implicated) if not what is said.[11] Among the more frequently discussed uses of specific indefinites are those that give rise to exceptional scope effects - to be examined below - and those that introduce individuals into discourse - as in (27). Arguably, in both of these cases the exploitation of the conversational principles is an efficient means to an end which is not easy to realise otherwise. Similarly, it is being argued here that this form of exploitation is an efficient means to getting across *at least* readings of NQNPs in many contexts.

In what follows, the reasoning behind the exploitation of specifics (including specific NQNPs) will be cast in Stalnaker's two-dimensional framework (see Stalnaker 1978, 1998). In that framework, the set of worlds in which what is common ground is true is called the 'context set'. Speech acts have the affect of changing the context set in two ways: The facts about the speech act can affect the context set and, once the speech act is accepted, this also results in a change to the context set. For example, when the speaker utters S in order to assert some proposition p, the effects include the elimination of alternatives in which p is not true in addition to the elimination of alternatives in which the speaker does not utter S. As a preliminary, it will be worthwhile to work through the account of the discourse in (27) in the two-dimensional framework. We will be using Schwarzschild's approach while treating NQNPs as bilateral quantified noun phrases.[12]

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question stems also from the idea that the meanings of expressions are not generally up for grabs in conversation.

[11] In 'Logic and Conversation' Grice (1975) reserves a special category of implicatures which turn on the open flouting of maxims. In these cases, the implicatures 'repair' the violation not at the level of what is literally said but at the general communicative level.

[12] At this stage, nothing precludes using Kratzer's approach to derive many of the readings to be discussed. However doing so would introduce an unnecessary ambiguity into NQNPs. This point will be taken up in the final discussion.

In the context described for (27), where it is assumed the speaker is introducing two indivduals into the discourse rather than making a claim about the totality of men walking in the park, one assumes that the noun phrase is specific. Adopting Schwarzschild's approach we can say that the NQNP's domain of quantification is implicitly restricted so that one collection of individuals remains. Just how implicit domain restriction arises is a matter of some debate (see Stanley & Szabo 2000 a.o.) but it seems clear that it is an independent phenomenon. As such, we follow Schwarzschild and simply assume that such restrictions occur. However, for the purposes of discussion in what follows, we will make this implicit restriction explicit along the following lines. Where a noun phrase like, *[two men]* is specific, we talk about its interpretation as if the form of the noun phrase is really *[two men who have P]* where *P* is a to-be-specified predicate. As the noun phrase is taken to be specific, it is presupposed that *P* applies to just one collection (of two men).

Given that the speaker has uttered the first sentence of (27) clearly intending the noun phrase to be understood specifically in this way, we can reason as follows: The speaker is violating the pragmatic principle that s/he express the same proposition in each context alternative. In fact, as far as can be presupposed, *P* could denote any singleton set containing a collection of two men and so the speaker's utterance causes the set of possible context alternatives to proliferate with different collections of two men in the denotation of *P* in different alternatives.[13] We can assume that, in flouting the pragmatic principle, the speaker is inviting us to see what follows on the assumption that the proposition she expresses is true. So at this point, we can repair the principle that the speaker express the same proposition in each alternative by assuming that in each currently live alternative the speaker expresses the proposition that what s/he says in that alternative is true. That is, in each alternative, the speaker expresses what Stalnaker calls the diagonal proposition. The diagonal is true at a world, w, if what the speaker says at w is true.

It is to be noted that, conceptually, the repair takes place *after* the utterance of the specific is made. That is, in updating the context with the diagonal proposition, we are considering alternatives which we discriminate in terms of different denotations for *P* - among other things.

So, let us consider two live context alternatives, w and w' in both of which m_1 and m_2 are the two men in the collection *P* picks out. The difference between w and w' is that in w, but not w', m_1 and m_2 were walking in the park. In updating the context with the diagonal proposition, we eliminate w' and keep w. But notice that in w', there could just as well

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[13] In fact, strictly speaking, the alternatives proliferate in a more fine-grained way than is suggested here since it is really properties that context has to identify in resolving for *P*. In the case at hand, it is not necessary to take this level of fine-grainedness into account. Later, in modal contexts, it will be.

have been two or more men walking in the park (just not both m_1 and m_2). So, in updating the context set with the diagonal proposition, we may be eliminating context alternatives where two men are walking in the park. However, if we individuate alternatives as they were prior to the speaker's utterance, for any two men, m_i and m_j, we retain every alternative in which m_i and m_j walk in the park. This means that the overall effect is *almost* the same as updating the pre-utterance context with the existentially closed proposition given in (30):

(30) $\exists X[|X| = 2 \wedge men(X) \wedge walking\_in\_the\_park(X)]$

Strictly speaking, though, in the context of the utterance having been made (with alternatives now discriminated according to different values for *P*), the diagonal proposition in fact entails (30). In some cases, this difference may not be so important since the distinction between alternatives that turn on choices of values for *P* may not be so relevant and indeed the existentially quantified proposition may be taken to be the main point of the utterance.

So far so good. We have an account of how, for instance, the Kadmon examples come out seeming to involve unilateral interpretations of the NQNP. In fact, the account just given recapitulates in two-dimensional terms the more informal line of reasoning sketched in section 5 for this kind of example.

We begin to depart somewhat from Kadmon's favoured dynamic treatment of these cases when we look at pronominal anphora. Let us continue with the second segment of (27), which contains the pronoun, 'they'. In line with the assumptions in Stalnaker (1998) we could assume that such expressions are simple variable terms of direct reference, which come with a presupposition that their referents are uniquely salient.[14] At the point where the second utterance is made, we currently have active alternatives in which the collection of two men in the denotation of *P* was walking in the park. Taking the pronoun to be referring to this collection, we update the context by eliminating alternatives where the men in question are not whistling. The result is a kind of extended diagonal proposition which entails (31):

(31) $\exists X[|X| = 2 \wedge men(X) \wedge walking\_in\_the\_park(X) \wedge whistling(X)]$

Again, if we do not care too much about what *P* expresses, we may understand the speaker's main point to be this implication of his or her utterance. It may seem that, in this way, we derive an interpretation which is virtually equivalent to that which is derived by simple dynamic binding. However, Stalnaker's assumptions about the pronoun 'they' are

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[14] The motivation for the presupposition of unique salience comes from examples like, 'John is married. ?She is French' and Partee's notorious marble example.

too weak.[15] To see this, consider that the following discourse is infelicitious:

(32)   Two boys were playing cricket in my garden. # He hit a shot and broke one of my windows.

If we assume that in each alternative, 'he' in the second utterance simply refers to a uniquely salient individual, there is nothing to stop us individuating alternatives whereby one of the two boys mentioned in the first utterance is uniquely salient and other alternatives where the other is uniquely salient. The resulting interpretation would be the same as having said, 'One of them hit a shot...'. Given that the continuation would have been otherwise perfectly coherent it seems we must conclude that the pronoun carries with it more than simply the presupposition of unique salience. It seems indeed that the definiteness of the pronoun brings with it some kind of additional indentifiability constraint. One apparent way to satisfy that constraint is by identifying individuals as those the speaker has in mind in uttering a given noun phrase. In the case we are considering, the utterance of 'two men' in the first segment is understood to be traceable via a causal chain to a collection of two men whose activities serve as the grounds for the speaker's utterance. To say that the individuals are identified in this way is to say that P is identified with a token-reflexive property 'being the collection s had in mind in producing u' (where u is the utterance of the noun phrase). Having identified P in this way, the distinction between context alternatives is no longer irrelevant as it is in the Kadmon example and thus the strengthened proposition is taken to be expressed, i.e. that the collection of individuals that s had in mind in producing u whistled.

That this is an acceptable way of identifying individuals is motivated by the pronominal contradiction examples, like (28) above, where one can talk about the specific individuals introduced by dicourses such as (27) independently of the descriptions used by the speaker (see van Rooy 2001 and Breheny 2004 for more evidence of this kind).

6.2   More Non-Specific Specifics and the Pragmatics of Existential Closure

Anyone familiar with the literature on specific indefinites and exceptional scope will recognise our treatment of the Kadmon-style examples above as applicable to often-discussed cases of exceptional scope indefinites. Consider for instance (33):

(33)   Mary will accede to the throne if two old uncles die before she does.

An approximate gloss of the relevant reading of this example is that there are two old uncles who are such that Mary accedes if they die before she does. The example is referred to as a case of exceptional scope since it is assumed that the reading in question

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[15] This point is developed at greater length in Breheny (2006a).

ought not to arise if normal grammatical constraints on scope are functioning. Most treatments of this kind of example aim to derive the relevant interpretation while assuming that there is no movement of the noun phrase out of the subordinate clause (see Reinhart 1997, Kratzer 1998 a.o.). Such examples are discussed in Breheny (2006b) where the no-movement view is taken and where it is shown that these cases can be handled via diagonalisation. In the case at hand, even if we assume that 'two old uncles' is interpreted bilaterally, it is a straightforward matter to account for the understanding in question if the NQNP is understood specifically (i.e. as '[two old uncles who have P]') and it is assumed the speaker intended to convey the diagonal proposition - as above. The result after updating with the diagonal is that, in each context alternative, the two old uncles in the denotation of P are such that their death leads to Mary's accession. Given that we have included no other information about P in our update, we can see that the overall effect of the utterance is to inform us that there is a way of choosing a pair of old uncles such that their death prior to Mary's leads to her accession.

It is relevant at this stage to note that, on the diagonalisation account of exceptional scope examples like (33), it is not necessary to assume that the speaker is communicating that it is two old uncles *s/he has in mind* that stand between Mary and the throne. However, in contexts where it is relevant, it may be that the speaker is optionally conveying this extra information about whoever satisfies P and this information can be accommodated. Without this extra assumption, diagonalisation would have the effect that the simple existentially closed proposition is being conveyed. In Breheny (2006b) it is argued that it is a virtue of the diagonalisation account over contextualist accounts such as Kratzer's that it accommodates the intuition that sometimes examples like (33) can be understood in a purely existential way.

It is also relevant to note at this point that other types of noun phrases that are bilateral can give rise to the same understanding:[16]

(34) a. Mary will accede to the throne if just two old uncles die before she does.
      b. Mary will accede to the throne if exactly two old uncles die before she does.

Note that, according to the *non-specific* construal of (34a,b), Mary's accession is triggered by the death of any two old uncles, so long as there are no more than two. This is not the same as the specific 'exceptional scope' readings of these examples.

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[16] It should be granted that (34b) in particular requires a little context to justify the use of 'exactly', but not too much. For instance, one can easily get the reading in question if (34b) is a response to a vague claim about the accession, for instance that there are 'not many' people standing between Mary and the throne.

That these exceptional scope facts fall out of the same account as that given for the Kadmon-style examples and the dynamic binding cases adds support to the proposal that apparent unilateral, existential understandings of NQNPs are derived in this way, starting with bilateral NPs understood specifically.[17] But there is more motivation that this general pragmatic approach to deriving existential closure is the correct one once we look at a popular alternative involving a mechanism of *free* existential closure.

As mentioned above, a popular analysis of indefinites involves the use of choice functions. Building on earlier work in Reinhart (1997) and Winter (1997), recent work (see Winter 2004 and Schlenker 2006) has seen the need to interpret indefinites using choice functions of variable adicity - along the following lines (where # is an object of which no predicate is true):

(35) $F_{<n>}$ is an n-ary General Skolem Function if for any n-tuple $<d_1, ..., d_n>$ of objects and any set E,
        $F(d_1, ..., d_n, E) \in E$ if $E \neq \varnothing$
        $F(d_1, ..., d_n, \varnothing) = \#$ if $E = \varnothing$ (Schlenker 2006, p. 288)

For reasons discussed in Kratzer (1998) and elsewhere, examples such as (36a) below need to be analysed using a unary General Skolem Function whose domain is the set of linguists. This analysis is represented in (36b):

(36)   a.   Every linguist has studied every analysis that has been proposed for some problem.
       b. $\exists F_{<1>}[[$Every linguist$]_i$ [has studied every analysis that has been proposed for $F(x_i,$ problem).

While Kratzer (1998) leaves the skolemised choice function variable free, Schlenker (2006) and Winter (2004) both advocate a mechanism of free existential closure of such free variables. Important motivation for this move can be found with examples such as (37a,b), discussed in Chierchia (2001):

(37)   a.   Not every linguist has studied every analysis that has been proposed for some problem.
       b.   No linguist has studied every analysis that has been proposed for some problem.

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[17] The connection between 'dynamic binding' and exceptional scope cases is made in van Rooy (1998). That paper builds on van Rooy (1997) and explores the issues of implementing the pragmatic technique of diagonalisation into a dynamic semantics for a formal language and does not explore the case of NQNPs.

Chierchia argues against Kratzer (1998) that (37a) need not be understood as if the skolemised choice function variable has been given a particular referent by context (as shown in (38a) below) but as if some kind of existential closure has occured immediately within the scope site of the binder of the implicit variable of the function - as represented in (38b):

(38) a.   [[Not every linguist]$_i$ [has studied every analysis that has been proposed for F(x$_i$, problem)]]

      b.   [[Not every linguist]$_i$ $\exists F_{<1>}$ [has studied every analysis that has been proposed for F(x$_i$, problem)]]

This is not to say that (37a,b) cannot be understood as if there were a particular kind of problem in question - as where the indefinite is modified with 'a certain'. But it also seems clear that Chierchia's point stands that these examples can be independently understood in the manner suggested in (38b) where there are potentially different kinds of problem for different linguists.

So it seems that some mechanism of local existential closure is required for these examples. However, in Breheny (2006b) it is argued that the nature of that mechansim is pragmatic and derives from the same kind of reasoning (using diagonalisation and accommodation) as employed in the examples already discussed. Below, we will see how this kind of account can be extended to derive the intermediate-scope existential closure reading of (37a) glossed in (38b). But first we should note that any account positing a mechanism of free existential closure (such as Winter's or Schlenker's) has problems of its own in that it over-generates. This problem is discussed at length in Schwarz (2001, 2002) and can be illustrated using the following example:[18]

(39)   Exactly two students read every paper that some professor wrote.

While one can imagine contexts where (39) is understood according to the intermediate-scope gloss in (40a), there is no reading of this example that corresponds to widest-scope existential closure - represented in (40b) below - which is equivalent to (40c) (see Schwarz 2002 and Breheny 2006b for details):

(40) a.   [[exactly 2 students]$_x$ $\exists F_{<1>}$[x read every paper that F(x, λu.professor(u)) wrote]]

      b. $\exists F_{<1>}$[[exactly 2 students]$_x$ [x read every paper that F(x, λu.professor(u)) wrote]]

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[18] In this example, the noun phrase 'exactly two students' should not be understood specifically but rather it should be assumed that the domain of quantification (of students) is given - say the students in the course the speaker is teaching.

c. [At least two students]$_x$ [[some professor]$_y$ [x read every paper that y wrote]] and [At most two students]$_x$ [[every professor]$_y$ [x read every paper that y wrote]]

The general observation in Schwarz (2002) is that the mechanism of free existential closure of a choice function variable generates unattested readings where closure is allowed at the root-clause level in all cases except where the binding quantifier is monotone increasing - as in (36a).

In Breheny (2006b) it is argued that the effect of intermediate existential closure can be achieved pragmatically and that the manner in which this effect is derived precludes the widest-scope closure reading.[19] The account can be illustrated by considering (41a), which has an intermediate scope reading - glossed in (41b):

(41) a. It's not true that Mary will accede to the throne if two old uncles die before she does.

b. It's not true that $\exists F_{<0>}$ [Mary will accede to the throne if F(two old uncles) die before she does]

As before, we procede by assuming that the NQNP is specific as suggested in (42) and following the diagonalisation path:

(42) It's not true that Mary will accede to the throne if [two old uncles who have P] die before she does.

The mechanism that gives rise to the intermediate scope effect involves accommodating a further assumption about *P*. Recall that in the discourse example (27), the definiteness of the anaphoric pronoun triggered a further accommodation about the identity of *P* - that it applied to the collection of individuals that the speaker had in mind. Recall also that such an accommodation can be optionally made for examples such as (33). Now, by accommodating a different kind of assumption about *P*, one can derive the intermediate scope effect for (41). The accommodation in question is that *P* picks out a 'first among equals' collection of uncles; to wit, a collection of two uncles which are such that if any pair of uncles' deaths trigger Mary accession, the deaths of the collection of two uncles

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[19] Recall that we derived the widest-scope readings of examples such as (33) via diagonalisation but that reading was derived as a relevant implication of the diagonal proposition. Recall also that the diagonal proposition was not equivalent to the existentially closed proposition but entailed it. In cases where the indefinite is in the scope of downward-entailing or other non-monotone quantifiers, the un-enriched diagonal would still entail the existentially closed proposition but that implication is generally not so relevant and not considered a reading by informants.

P picks out trigger her accession.

In general, if we have the NQNP in the scope of some operator, $[O_1...[$two Fs who are P]...], and this whole construction is in the scope of another operator, $[O_2...[O_1...[$two Fs who are P]...]], then we can always accommodate a 'first among equals' assumption about *P* so that we can have the intermediate scope effect without movement. Suppose ϕ is the result of extracting the NQNP from within the scope of the operator, O_2. That is, if the interpretation of $[O_2...[O_1...$two Fs who are P...]] is represented as $O_2'(\psi_1(\text{two\_Fs\_who\_P}'(\psi_2)))$ where two_Fs_who_P' is an $(((e,t),t),t)$ operator and ψ_2 is possibly null, then ϕ is $\lambda X[\psi_1(\lambda Q[Q(X)](\psi_2))]$. In that case, and where *P* is type $((e,t),t)$ the 'first among equals' presupposition is as follows:

(43) $\neg(\exists X[|X| = 2 \land Fs(X) \land \phi(X)]) \lor \exists X[P(X) \land \phi(X)]$

In the example under discussion, ϕ is $\lambda U[$Mary will accede to the throne if U die before she does]. Then, given that *P* is presupposed to pick out just one collection of two uncles, the assertion of (42) contradicts the right disjunct of (43) leaving us with the left disjunct of (43) being implied.

A similar story can be told for the derivation of the reading (44a) represented in (44b). One difference lies in the nature of the implicit modification. Again we follow Schwarzschild (2002) in observing that, generally speaking, one quantified noun phrase's domain of quantification can be dependent on another. As this is an independent phenomenon we can, for the purposes of this discussion, simply assume that the implicit restriction can be functional and is so in this case. Thus (44a) is understood as if the form were more like in (44c) where *P* is understood to pair each student in the domain of quantification with just one collection (of two professors):

(44)   a.   Not every student read every paper that two professors wrote.
       b.   [Not every student]$_i$ $\exists F_{<1>}$ [read every paper that F(x_i, two professors) wrote]
       c.   [[Not every student]$_i$ [read every paper that [two professors who have P(x_i)] wrote]]

The relevant 'first among equals' accommodation is given in (45):

(45) $\forall u[\text{students}(u) \rightarrow \neg(\exists Y[|Y| = 2 \land \text{professors}(Y) \land \forall y \in Y[\forall z[\text{paper}(z) \land \text{wrote}(z)(y) \rightarrow \text{read}(z)(u)]]]) \lor \exists X[P(X)(u) \land \forall y \in Y[\forall z[\text{paper}(z) \land \text{wrote}(z)(y) \rightarrow \text{read}(z)(u)]]]$

Given (44c) and (45), we learn that not every student read every paper that the collection of two professors paired with them by *P* wrote; and in addition that *P* pairs each student, x, with a collection of two professor all of whose papers x has read, if there is any such

collection. Thus, together, (44c) and (45) tell us just what (44b) tells us.

At this stage, one could ask what would cause anyone to make such an assumption as in (43) or (45). One answer is simply that there is no other way to obtain the relevant scope reading due to the above-mentioned grammatical constraint on scope relations. But more to the point, the answer lies in the fact that the pragmatic reflexivisation process is quite general. It *is* the discourse-level existential-closure mechanism and one good reason for thinking so concerns the fact that the effect of the unattested widest-scope existential closure cannot be obtained in this way. To see this, consider how we might try to extend the scope of φ to include also negation. That is, consider what happens if φ is λU[It's not true that Mary will accede to the throne if U die before she does]. If we plug this into (43), given that *P* is presupposed to pick out just one collection of two uncles, we get a disjunction whose right disjunct says the same as what is asserted while the left disjunct merely contradicts what is asserted. So the 'first among equals' presupposition adds nothing in this case. Similar considerations apply to (39) and (44c) (see Breheny 2006b). So, seeing discourse-level existential closure in this pragmatic way helps us to understand why the unwanted readings generated by free-closure are not available.

6.3   *At least* readings in modal and other embedded contexts

We can now return to some of the other examples discussed in the last section which seemed to motivate an ambiguity account:

(24)   a.   I predict twenty people will be there tonight.
           b.   I bet that five women finish in the top one hundred in this year's marathon.

(25)   a.   You must take two cards.
           b.   To qualify for this course, you must have two A grades.

It should be clear at this stage how the account of the narrow scope existential readings of these kind of examples ought to go. There is one refinement that needs to be made to the way we have been thinking about diagonalisation so far. In cases where the NQNP is in the scope of modal expressions it matters what property *P* expresses. Thus what we need to consider are context alternatives that differ according to which property *P* expresses. So to take (25a) as an example, the relevant first-among-equals assumption about *P* would be that, given a context alternative, w, *P* expresses a property, P_w, which is such that in each permissable alternative accessible to w, either there is no collection of two cards you take, or you take the collection that P_w picks out. Thus, in updating the context with the diagonal proposition, we retain the context alternative, w, just in case you take two or more cards in each deontic alternative accessible from w.

An obvious candidate for *P* in (25a) would be something like, 'being the first two cards

chosen' but in other cases, such as (46), a more general first among equals presupposition is required:

(46)   You must not take two cards.

So, considering the general class of modal contexts exemplified in (24) and (25), we can give the general scheme for the first-among-equals presupposition in (47) where w is a context alternative and P_w is the property expressed by *P* in w and R is the relevant accessibility relation for the modal:

(47) $\forall w'[w'Rw \rightarrow (\neg\exists X[|X| = n \wedge F(X) \wedge G(X)] \vee \exists X[P_w(X) \wedge G(X)])]$

Here G represents the scope of the existential closure. For both (25a) and (46) this would be $\lambda X.$You take X.

Note that with the relevant first-among-equals accommodation, an utterance of (25a) could be compared with an alternative utterance of 'You must take three cards' in terms of informativeness. Under these conditions, the un-uttered sentence would have been more informative and, if it would have been relevant, a quantity implicature to the effect that you need not take three cards would follow.

Hopefully, it should be clear by now that in following through the pragmatics of specificity and diagonalisation, we can account for all of the various *at least* readings of NQNPs discussed.

7.   Specificity, Diagonalisation and Anaphoric Relations

In this section, we will explore what kinds of anaphoric relations can exist between NQNPs and pronominals. We will see that where the NQNP antecedent has the *at least* reading, the approach being promoted in this paper gives a better account of the facts than any alternative account which just says that NQNPs are ambiguous. To get things started, let us return to the discourse in (27):

(27)   Two men were walking in the park. They were whistling.

We have seen how to derive the relevant interpretation of this discourse by starting out with a specific bilateral NQNP. It was also mentioned that there is good reason to think that the pronoun in the second sentence forces an identifiability condition, satisfied in this case by thinking about the possible specific collections of individuals as those the speaker has in mind. It is this extra dimension of the interpretation of such discourses that the simple dynamic binding account misses. Problems for dynamic binding multiply when

we consider that exceptional scope indefinites can give rise to similar anaphoric relations:

(48) Mary will accede to the throne if two uncles die before she does. Fortunately for Mary, they are very old.

Simple dynamic binding as discussed in DRT and elsewhere precludes binding from within the antecedent of a conditional. So, one option here for a dynamic unilateral account might be to follow Geurts (2000, *forthcoming*) in thinking about exceptional scope in terms of presupposition projection. In Geurts' favoured DRT framework, this would mean introducing a discourse referent plus the condition that it denote a collection of two uncles outside of the scope of the conditional. This discourse referent then would be accessible for the discourse referent introduced by the pronoun in the second sentence. But note that this approach still faces the problem of pronominal contradiction. For instance, an interlocutor could follow up (48) with (49):

(49) They are not uncles. They are second cousins.

This is handled in a straightforward manner by the diagonalisation account: At the point where the second utterance of (48) is made, in each alternative we have a collection of two uncles in the denotation of *P* and Mary accedes if these die before her. As with the discourse above, the definiteness of the pronoun forces one to identify the collection in question independently - the most natural way being to think about them as the collection the speaker has in mind. We then understand the correction in (49) as an injunction to alter what is presupposed, so that the collection the speaker has in mind are second cousins not uncles.

As an alternative implementation of the unilateral analysis of NQNPs we could think about them along the lines of the choice function account whereby the argument NQNP consists of a variable over choice functions as a sister to the plural predicate, $\lambda X.|X| = 2 \wedge \text{uncles}(X)$ and there is free existential closure of this choice-function variable. Trying to make this quantification dynamic would run into the same problems as before since dynamic binding of a choice function variable would require that 'they' in the second sentence of (48) have an implicit predicate. This predicate would have to be recovered from its antecedent (possibly along the lines of Elbourne 2005). But then the pronominal contradiction cases become difficult to handle. As a non-dynamic alternative, one could suppose that in uttering the first sentence of (48), the speaker makes it clear that s/he has a particular choice function in mind and hence a particular collection of individuals in mind. This collection could then be the referent of the pronoun in the second sentence. So, there is a way to account for the anaphoric relation in (48) via this modified unilateral account of NQNPs (using choice functions and free existential closure). But, being static, this account misses some essential properties of dynamic binding which the diagonalisation approach retains. To see this, let us return to a Kadmon example.

Consider a case where a building superintendent is asked by a staff member about getting four chairs needed for a meeting. The superintendent replies as follows (knowing there to be many more than four indistinguishable chairs in the next room):

(50)   There are four chairs in the next room. You can borrow them overnight but if you return them after 10am, you will be charged.

Considering Kadmon's favoured dynamic treatment of this example, we could envisage the construction of a Kamp-style Discourse Representation Structure where the discourse referent introduced for the first sentence is identified with the DR in the antecedent of the final conditional (see Kadmon 2001 and Kamp & Reyle 1993 for suggestions of how this could be done). But note that the resulting structure says something weaker than the full force of the super's reply. The DRS would only tell us that there are four chairs in the next room which are such that if you return them late you will be charged. However, the force of the super's utterance is that *whichever* four chairs you choose, they are such that if you return them late you will be charged.

We can get a good account of this example by considering the diagonalisation treatment: After the first sentence is uttered and the context updated, we are faced with context alternatives in each of which a collection of four chairs is the denotation of the implicit restriction of the NQNP and these four chairs are in the next room. The second sentence contains a pronoun which has a salience presuppposition, hence, in each alternative, the pronoun denotes the set of four chairs in the denotation of the specific's restriction. But the pronoun also has an identifiability condition. In this case, we cannot satisfy the condition with the property of being the collection the speaker has in mind (because the speaker has no particular chairs in mind). But we can satisfy the condition with the property of being the four chairs that the addressee takes in realising her goals. We can get a sense that this is the kind of description that most readily comes to mind by noting the mild infelicity of the following alternative reply by the super where we interpret 'four chairs' as before, in the *at least* way (i.e. it is clear the super has no particular chairs in mind):

(51)   There are four chairs in the next room. You can borrow them.

On the account under consideration, the reason why this seems strange is that we want to understand 'them' as 'the four chairs you borrow' and yet then the utterance would be taking for granted that for which permission is being given.

Having understood the second utterance of (50) as just suggested, the correct understanding of the conditional sentence follows automatically: we eliminate alternatives where the audience does not return the four chairs they borrow before ten and do not get charged.

The choice function-plus-free-closure account that we are still considering would have severe difficulty deriving this reading without making use of the style of reasoning that is employed in the alternative two-dimensional account being offered here. This is so because it is in the nature of these Kadmon-style contexts that no determinate set of chairs becomes salient for future reference. To see this, let us suppose that the first sentence could be analysed along the following lines:

(52) $\exists F_{<0>}[\text{in the next room}(F(\lambda X.|X| = 4 \wedge \text{chairs}(X)))]$

The definite pronominal in the second sentence requires both a salience condition and an identifiability condition to be met. Recall that, for some previous examples, to meet the salience condition it would have sufficed to suppose that the speaker intends to raise the objects s/he has in mind to salience. However, in this case, the speaker does not have a set of four chairs in mind and so there is no particular collection of four chairs being raised to salience in this way.

An alternative idea might be to assume that the noun phrase 'four chairs' is implicitly restricted in some way and the pronoun in the second setence is understood as an E-type - i.e. it denotes the total set of chairs in the next room under discussion. The natural restriction for this example would be along the lines of 'There are four chairs you can take in the next room'. Thus the pronoun in the second sentence could be understood as, 'the four chairs in the next room you can take'. However, there could just as well be more than four chairs in the next room that you can take. So this way of restricting the NQNP still does not single out four chairs four future pronominal reference. It seems only one kind of restriction would work in this case and that would be along the lines of 'there are four chairs you will take in the next room'. But to put this restriction into an existential claim would mean taking for granted that you do take four chairs. In two-dimensional terms, this means that in each context alternative, there is a particular set of four chairs the audience takes. In other words, to get the anaphora facts right in this case, one has to take for granted that there are four chairs that you take under discussion. Given that this same form of reasoning can be employed to deliver the effect of free-existential closure and without the need to posit a formal ambiguity in NQNPs then it seems clear that the univocal bi-lateral account is conceptually more appealing here.

To sum up this discussion: By considering relations between NQNPs with *at least* readings and pronominal anaphora we reveal again that the apparent existential closure effect comes with a form of pragmatic reasoning whereby we take for granted that there is a specific collection at issue and see what follows from that. This form of reasoning makes this specific collection available for subsequent pronominal reference. Thus, close consideration of the anaphora data further motivates the idea that when NPs are used in this specific way, the discourse becomes implicitly modal - in the manner that Stalnaker's two-dimensional framework supposes.

8   Discussion

Let us compare what is being proposed with a modification of Geurts' ambiguity account. On this modification, 'two students' can be understood as a predicate expression (possibly, as Geurts 2006 suggests, due to a regular type coercion from the bilateral noun phrase meaning). Predicate nominal expressions (like 'brown cows') can find their way into syntactic argument position as sister to a (possibly skolemised) choice function variable. Mandatory existential closure ensures that [brown cows] or [two cows] have existential force but (in line with the free-existential-closure account) existential closure does not necessarily occur at the level of the noun phrase nor does it have to occur at the level of the root clause. So far we have come across three kinds of problem with this account. Firstly, if we follow this reasoning we should admit that 'exactly two' and 'at most two' have *grammatically* derived existential monotone readings (respectively, *at least two* and *at least one*)[20]. Secondly, as discussed in Section 6, the account generates unattested wide-scope existential readings where the NQNP is in the scope of a non-upward monotone quantifier. Thirdly, as just discussed, anaphoric relations are problematic for this account.[21] It seems plausible on the other hand that this otherwise mysterious existential closure mechanism is really just the pragmatic reflexivisation process. Such a process accounts not only for exceptional scope facts but for cross-sentential anaphoric facts and it naturally accounts for all of the *at least* readings of NQNPs that appeared to challenge the bi-lateral account.

An alternative ambiguity account would suggest that the unilateral version of 'two students' is dynamic and exceptional scope is a matter of presupposition projection as formally specified in van der Sandt (1992), Geurts (1999) and elsewhere. As Geurts (2000) admits, such an account of exceptional scope re-creates noun phrase movement at the level of discourse representation. As such, it faces the problems raised in Schwarzschild (2002) concerning its insufficient specificity. For example, a simple wide-scope analysis of (53a) below, as represented in (53b), is much weaker than the intended interpretation since it would only take one obscure poem by Pindar that no student recited for it to be true:

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[20] Of course, the account defended here makes it in principle possible that 'exactly two' has a *pragmatically* derived existentially closed reading. This issue will be taken up shortly.

[21] A fourth problem for this account stems from the fact that bare plurals, like 'brown cows', tend to resist exceptional scope readings (although such readings are perhaps not impossible).

(53)   a.   John gave an A to every student who recited a difficult poem by Pindar.
       b. $\exists x[\text{Pindar\_poem}(x) \land \forall y[\text{student}(y) \land \text{read}(x)(y) \rightarrow \text{John\_gave\_A\_to}(y)]]$

In addition to this type of problem, we have seen that dynamic binding as found in DRT and elsewhere does not capture many of the subtle facts which are brought to light by the diagonalisation account. In fact, following from Stalnaker (1998) and van Rooy (2001), the facts here suggest that simple dynamic binding rather imperfectly recreates what goes on at the pragmatic level when anaphoric relations between indefinite noun phrases and pronominal anaphora are being established.

Once we consider the variety of cases in which *at least* readings of NQNPs surface, it seems very plausible then that they could well be dealt with as cases where a bilateral NQNP is understood specifically and the relevant pragmatic reasoning follows. The phenomena we have looked at here have manifestations independently of NQNPs, occuring with all kinds of indefinite or 'weak' noun phrases (see Breheny 2006b for a discussion).

The fact that NQNPs pattern in the same manner as other non-definites with regard to exceptional scope allows us to make some observations about the merits of analysing indefinite noun phrases generally as singular terms consisting of a choice-function variable as sister to a predicate. On the one hand, we could reserve this analysis for specifically used noun phrases only and suppose that non-specific indefinites are quantificational. In that case, one could argue that Schwarzschild's singleton indefinite proposal coupled with the diagonalisation account of apparent existential closure obviates the need to posit this formal ambiguity. On the other hand, we could suppose that all indefinites are singular terms and that free existential closure can be invoked to handle non-specific cases as well as exceptional scope cases. In addition to the problems for this line of thinking already mentioned here (and in Schwarz 2002), there is good reason to think that non-specific NQNPs really are bi-lateral non-monotonic quantifiers (see the discussion around (6a-d)). As such, NQNPs would not be subjectable to this uniform analysis. But then, as non-definites, NQNPs seem no different from other indefinites in terms of their potential for exceptional scope and specific readings. The 'singleton indefinite' account does make a uniform analysis of NQNPs possible and this fact provides another reason to think about non-definites generally as quantifiers and not singular terms.

At this stage, a few words are in order about the modified numerically quantified noun phrases 'exactly two', 'just two' and so forth. According to what has been claimed so far, these noun phrases are understood as quantifiers with the same meaning as the unmodified NQNP (as suggested in (29)). This being the case, one could legitimately wonder why these noun phrases resist *at least* readings even though such readings ought in principle be derivable via the same process as for the unmodified case. Why for

instance, in Kadmon-style contexts, the building super's saying, 'There are exactly four chairs in the next room' cannot be understood in the same way as his saying 'There are four chairs in the next room'?[22]

In answering this kind of query, one should first note that *at least* readings of these modified NQNPs are available in exceptional scope contexts. Consider again (34a,b) repeated below:[23]

(34)   a.   Mary will accede to the throne if just two old uncles die before she does.
        b.   Mary will accede to the throne if exactly two old uncles die before she does.

It is to be conceded that (34a) has a more accessible exceptional scope reading than (34b). But one could argue that this has to do with how easy it is to see a reason for modifying the NQNP: 'just two' emphasises perhaps how small the number is, how close Mary is to the throne. The suggestion in the text above was that the exceptional scope reading for (34b) would be more prominent if 'exactly' was clearly being used to emphasise the precision of the claim. For example, if (34b) is a response to someone vaguely mentioning that 'one or two' or 'a few' relatives stand between Mary and the throne, then the specific, exceptional scope reading becomes more prominent - especially where the modifier, 'exactly' or 'two' is stressed.

These considerations suggest the beginnings of an explanation for why these modified NQNPs normally resist the *at least* interpretation in many other contexts: If the semantic rules of English determine that 'two students' and 'just two students' or 'exactly two students' contribute the same function to the determination of truth conditions, then the use of 'just' or 'exactly' ought to be otiose - since people prefer to express a given semantic content in as few words as necessary and they prefer not to have to parse unnecessary constituents. In fact, if one used the modified form instead of 'two' for no

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[22] Note that this dilemma for the pragmatic account of the derivation of *at least* readings is similar to that raised for those accounts that derive the *at least* readings via a systematic, grammatically determined existential closure process. However it is not as severe in the current case since the *at least* readings are pragmatically derived and therefore have to be justified according to rational principles of conversation. In the grammatical case, the readings in question just ought to be available unless blocked by a further rule.

[23] Note that, as with (33), the exceptional scope reading of examples like (34a) is really *at least* reading. To see this, we should modify the example so that it reads 'Mary will accede to *a* throne...'. With the NQNP specific, the speaker is understood to be talking about Mary's accession via the death of a particular collection of uncles. It does not preclude Mary standing in more than one line of accession with yet more old uncles preceding her.

reason, one would be violating a Gricean maxim enjoining brevity. Thus the use of 'just' or 'exactly' ought to prompt one to find a reason for that usage.

Now it seems that 'just' and 'exactly' have different shades of non-truth-conditional meaning that makes their use conditions slightly different. For example, 'just flat' and 'exactly flat' cannot be appropriately used in the same set of situations. 'The countryside for the cycle race was just flat' seems to presuppose that there was an expectation that the countryside was otherwise (in parts at least). 'The countryside for the cycle race is exactly flat' suggests a degree of precision in the statement that would not have been conveyed had the modifier not been present. As suggested in Lasersohn (1999) 'exactly' seems to function as a 'slack regulator' in these cases. Lasersohn's idea is that predicates like 'flat' are often applied to referents of which they are not strictly true. Although the resulting assertion is literally false, Lasersohn observes that the utterance is judged acceptable up to contextually defined limits of lassitude. The function of a slack regulator, according to Lasersohn, is not to affect truth conditions but to narrow down the degree of acceptable lassitude.

On the question of whether 'exactly' functions as a slackness regulator for NQNPs, Lasersohn claims that it does so only in the case of telling the time. He suggests that generally speaking, 'exactly'-modified NQNPs do not have the same literal truth-conditional content as unmodified NQNPs. If in contrast to Lasersohn we accept the arguments in this paper that unmodified NQNPs are already bilateral, we can consider whether 'exactly' ever functions as a slackness regulator for NQNPs in non-time-telling contexts - and the answer of course is that they do. Consider, for example the following instructions in a recipe:

(54) a.   Take 200g of butter.
      b.   Take exactly 200g of butter.

One can imagine that the instruction in (54a) allows for different degrees of lassitude depending on whether the recipe is for a fine pastry (low degree of slack) or a butter sauce (higher degree of slack). Accordingly, the modification by 'exactly' in (54b) has different slackness regulation effects in the different examples.

Another typical case where 'exactly' would function as a slackness regulator would be in the case of round numbers. As Krifka (2007) observes, round numbers are very often used in an approximative way. (55a) below would often be acceptable if one or two students more or less than twenty were taking the class. Thus (55b) forestalls such pragmatic loosening:

(55) a.   Twenty students are taking the advanced syntax class.
      b.   Exactly twenty students are taking the advanced syntax class.

It is also well-known that using a non-rounded number often has the opposite effect - that of being precise. Where the context-question is 'How many students are in the advanced syntax class?', (56a) below is understood as being as precise. But if there is no need for slackness regulation in this case (56b) ought to give rise to some other kind of effect, according to the pragmatic account of the function of 'exactly'-modification of NQNPs sketched above:

(56)   a.   Twenty one students are taking the advanced syntax class.
       b.   Exactly twenty one students are taking the advanced syntax class.

Indeed intuition suggests that the speaker of (56b) would be attempting to convey something else about the figure mentioned - for example, that s/he has obtained it on good authority or that s/he is using the official figure.

The fact that 'exactly' can function as a slackness regulator with NQNPs provides further indirect evidence for the bilateral account. For if, as Lasersohn supposes, the function of 'exactly' applied to cardinals is to make unilateal NQNPs bilateral, then it literally encodes a different function to his slackness regulator. Thus it is somewhat curious that 'exactly'-modification still functions as a slackness regulator in many cases. In addition, we have seen that modification of already precisely understood NQNPs (as in 56b) drives the search for further effects. This is to be expected on the Manner maxim-based account of NQNP modification but somewhat mysterious if we assume 'exactly' has truth-conditional significance when applied to cardinals.

We are now in a position to address the dilemma raised at the beginning of this discussion of modified NQNPs. Suppose that you tell the superintendent that you are looking for four chairs and he replies as in (57) below:

(57)   There are exactly four chairs in the next room.

You reason as follows: Given that the unmodified utterance would have sufficed to inform me that I can get the chairs I need from the next room, in using 'exactly' the super must have wanted to convey some information over and above what the unmodified utterance would have conveyed. While the unmodified utterance would have addressed my immediate concerns, it would not have given any information about whether there are four, five etc. chairs in the next room. Although this question is relevant only to a secondary degree, I can see no reason for the use of the modification other than to raise the salience of this question. I thereby assume that the superintendent wished to address this question in addition to my concerns. The utterance would address both of these issues if it was non-specific, therefore I conclude that it is the non-specific reading that is intended.

So the strategy for the bilateral-plus-diagonalisation account would be to consider contexts where the *at least* reading typically arises to see whether the effects of modification can be accounted for on pragmatic grounds. A straightforward case would involve modifications of examples like (6a). In these cases, the *or more* reading results from a background implicature and should persist even where the NQNP is modified. This seems to be right - especially in the case of (58b). (58c) perhaps requires a little more context in order to give the modifier some motivation. For example if the audience has just uttered: 'I have exactly two children' to the benefits officer:

(58)　　a.　　If you have two children, you qualify for this benefit.
　　　　b.　　If you have just two children, you qualify for this benefit.
　　　　c.　　If you have exactly two children, you qualify for this benefit.

For (58b,c) the *at least* readings can be derived via the background implicature given the non-specific reading of the modified NQNP.

By contrast to the above examples, in the context of necessary conditions we saw that the *at least* reading has to come via the specific reading plus diagonalisation. The account we are considering suggests that one ought to be able to derive *at least* readings for modified NQNPs - so long as there is an independent contextually established purpose for the modification. The following context seems to work fine for 'just' in (59a) but perhaps not so well for 'exactly' in (59b):

(59)　　a.　　Normally, at this university you have to have three A grades to qualify for entry to a graduate programme, but for this Linguistics Masters degree, you have to have just two A grades from your undergraduate course.
　　　　b.　　Normally, at this university you have to have three A grades to qualify for entry to a graduate programme, but for this Linguistics Masters degree, you have to have exactly two A grades from your undergraduate course.

However, a better context for 'exactly'-NQNPs would involve a typical use of this modification - such as slackness regulation associated with round numbers. Consider the following example with the additional background information that beach clubs are normally fairly lax in enforcing their own rules (such as for dress codes):

(60)　　In order for this beach club to recognise you as an official life-guard, you have to have attended exactly twenty training days on the beach under the guidance of a senior life-guard.

Obviously (60) does not preclude your official recognition if you do more than twenty days training.

Whether this would be the whole story about the difference between modified and unmodified numerals is an open question. While it is at least plausible that the difference between the two types of expression can be accounted for along these pragmatic lines, it may be that, in addition, the use of 'exactly'-modified NQNPs is associated with a kind of processing bias whereby the non-specific reading is made initially most prominent to the audience. The existence of such a processing bias would not rule out the *at least* readings for the modified form in principle but it would make such readings far less accessible - in the sense that they would require a lot more contextual support and inferential work to derive.

The point at which a processing bias transforms into a hard rule of grammar is perhaps not so easy to pin down and it is an open question whether in a given language, a marker such as 'exactly' actually blocks the specific-plus-diagonalisation reading. Such may be the case in Hungarian where NQNPs in focus position are apparently mandatorily understood according to the *exactly* reading (see Kiss 2007).[24][25]

The next point of discussion leads on from the question of modification and concerns monotonicity. Given that the bilateral account here is able to derive apparent *at least* readings by way of the speaker expressing a (possibly enriched) diagonal proposition, we do not have any problems predicting that there is a way to construe the following discourses in a coherent manner:

(61)   A: John ate three cookies.
        B: Therefore John ate two cookies.

(62)   A: Two students drank beer.
        B: Therefore, two students drank some alcoholic beverages.

(63)   A: Two male students drank beer.
        B: Therefore two students drank beer.

Note that, the bilateral-plus-diagonalisation account, as well as the ambiguity account, imply that the monotonicity inferences for NQNPs exemplified in (62-63) are acceptable only on the *at least* reading of B's utterance.

A number of commentators have suggested that data concerning negative polarity items raise potential problems for the bilateral-plus-diagonalisation account and for the

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[24] Thanks to Anna Szabolcsi for drawing my attention to the Hungarian data.

[25] However, see also the discussion in the next section of the possibility that in some languages (Hungarian being one of them), NQNPs in fact encode an *at least* meaning.

ambiguity account[26]. It seems that examples such as (64a) below have become accepted as felicitous in the NPI literature (see van der Wouden 1997, Ladusaw 1996). In contrast, (64b) is not regarded as acceptable:

(64)   a.   Exactly two students asked any questions at the workshop.
       b.   Two students asked any questions at the workshop.

Accepting the judgements as they stand (even though almost all native speakers of English I have interviewed have judged (64a) in addition (64b) to be of low acceptability) these facts are not necessarily problematic for the accounts we are considering. However, the accounts come into conflict with any proposal that the licensing condition on these weak NPIs is that they be in non-upward monotone environments. If this in fact was the licensing condition, then the unmodified NQNP should just as well license NPIs.

One reasonable response for either the ambiguity proponent or the bilateralist is to observe that the non-upward monotone proposal for (weak) NPIs looks to be very strong and not without problems of its own. For example, even informants who judge examples like (64a) to be potentially acceptable do not always judge NPIs to be acceptable in the scope of 'exactly'-modified NQNPs. As discussed in Rothschild (2006) large numbers tend to degrade the acceptability of NPIs:

(65)   ? Exactly two million people have ever visited this forest.

Rothschild also notes that there are other non-monotonic environments in which NPIs are clearly infelicitous. He cites (66a) but others such as (66b-c) are not difficult to find:

(66)   a.   ? An even number of students ever handed in their assignment.
       b.   ? Approximately five students ever handed in their assignment.
       c.   ? Precisely two pints of blood was ever taken from his body.

These data suggest that the condition that weak NPIs be in a non-upward monotone environment is at best a necessary condition and that other factors determine the acceptability of weak NPIs.

Without wishing to provide an exhaustive account for the differential judgements surrounding (64a,b) in this brief discussion, it is worth noting that informants are happier to judge the examples in (67a,b) as felicitous - where there are different modifiers on the NQNP:

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[26] In particular, I acknowledge an anonymous Journal of Semantics referee and Daniel Rothschild for putting this issue on the agenda for both the bilateral and ambiguity accounts.

(67)   a.   Just two students ever handed in their assignment.
       b.   As few as two students ever handed in their assignment.

The difference between the modifiers in (67a,b) and 'exactly' and 'precisely' seems to be that the former suggest more strongly an expectation that more students would hand in their assignments. In such contexts, they would also pattern with 'only' in emphasising or foregrounding the inference that no more than the number mentioned handed in their assignment. Just why these weak NPIs are acceptable in these environments is an open question but it does not seem unreasonable to suggest that their licensing requires the *overt* presence of an element which realises the relevant foregrounding function. It is possible that informants who judge (64a) to be acceptable can see 'exactly' being put to this use also. This possibility is also motivated by the small-number constraint.

9.   Summary and Conclusions

We started out by considering NQNPs as encoding solely a unilateral meaning and Gricean or neo-Gricean quantity implicatures as accounting for cases where these noun phrases have a bilateral, *exactly* reading. The evidence against this type of account that was reviewed also strongly suggests that it is more likely that NQNPs encode a bilateral semantics with the other readings being derived in some way from there. The main purpose of this paper has been to set out just what kind of pragmatic reasoning would be involved in deriving the *at least* readings if NQNPs were univocally bilateral. In addition, it has been argued that the reflexive-style of reasoning appealed to here is recruited independently, in mediating anaphoric relations between non-definite noun phrases and anaphora and in exceptional scope cases. In fact, this form of reasoning is in all likelihood ubiquitous in conversation, perhaps to an extent comparable to quantity implicature (see Perry 2001, Stalnaker 1978 for some further cases).

It should be noted however that there has been no argument in this paper that NQNPs *have* to have bilateral semantics, only that NQNPs in English encode a bilateral meaning and that the various *at least* readings are derivable via pragmatic reasoning. It is conceivable that in other languages the correlates of NQNPs are unilateral in some way and that they give rise to a different pattern of readings. For instance, Anna Szabolcsi (personal communication) points out that the pattern of readings found for English in (6a) and (6d) does not obtain in Hungarian. As also mentioned above, Kiss (2007) suggests that focussed NQNPs in Hungarian are mandatorily given *exactly* readings. So it may be that there is a different system in Hungarian and other languages for realising the *at least* and *exactly* readings. This is a matter for further research.

The main and obvious competitor to this proposal for English NQNPs is to say that they are ambiguous in some fashion. In the course of the paper, a number of versions of the

ambiguity story were encountered. The focus of Geurts polysemy story is on the fact that quantified noun phrases of a certain class can be coerced into to predicate expressions and predicate expressions can be coerced into existential quantifiers (using, respectively, *BE* and *A*). However, it was found that Geurts' appeal to the apparent regularity with which these coercions occurred was does not carry so much force since, if Partee's type-shift were really an independent phenomenon in the grammar - driving interpretation - and not merely a post-hoc description of what happens in specific cases, then 'exactly n' and 'at most n' ought also to be polysemous. In addition to these considerations, Geurts' proposal as it stands does not make the link between the *at least* readings of NQNPs, the cross-sentential cases and the exceptional scope cases. Other ambiguity accounts which manage to make this link (dynamic quantifier and free-existential closure accounts) were found to be wanting in other ways and, at best, merely offer different and orthogonal formal descriptions of the effects of the pragmatic reflexivisation process when it is recruited in the interpretation of specific noun phrases.

Acknowledgements

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