Signals & Systems for Speech & Hearing

Week 8

Impulse responses

What you know about so far

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<th>time domain (= time on the x axis)</th>
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The BIG idea

Fourier analysis

waveform $\rightarrow$ spectrum

Fourier synthesis

spectrum $\rightarrow$ waveform

LTI-system

Using homogeneity...

System is homogeneous so output will be $\frac{1}{3}$ of the amplitude value as well.
using time-invariance...

30

LTI-SYSTEM

System is time-invariant so delay will remain 3 ms

30

delay: 3 ms

using additivity...

System is additive, so result of the two output waveforms added up will be the same as the sum of the input signals after having gone through the system.

approximation of arbitrary input waves by a combination of rectangular pulses

What we can do already

If we know the system response to one single pulse, we can predict the output for a complicated set of pulses (constructed by addition of scaled and time-shifted pulses)
Approximation of arbitrary input waves by a combination of rectangular pulses

Preliminary summary

- Put a rectangular pulse through the system to see what you get.
- This allows the prediction of system output to:
  - pulses of the same shape but different height
  - pulses of the same shape presented at a different time
  - a sum of an arbitrary number of pulses of the same shape, but any height and at any time.
- If you can adequately approximate a wave by a sum like this, you’re done ...
  - but this is usually not the case.
- So find a better pulse to approximate waves with.

What would be a better pulse?

But pulses provide less energy as they become narrower, so output shrinks ...
In order keep the output signal of sufficient amplitude, we increase the amplitude of the input pulse as it becomes narrower... until at some point the pulse has no duration and its amplitude is infinitely high.

An infinitesimally narrow and infinitely high pulse of finite energy is known as an impulse. True impulses are only a mathematical concept and do not occur in real life.

Because any wave can be created by adding together an infinite number of impulses appropriately scaled in amplitude and shifted in time...

Knowing the impulse response of an LTI system means that the output of the system can be predicted to any input.

Therefore, LTI systems are completely characterised by their impulse response (time-domain characterisation).
Another input wave: a single cycle of a sinusoid

What's the relationship between the impulse response and the frequency response?

- Knowing either the impulse response or the frequency response are sufficient to completely characterise a system...
- so they must contain the same information...
- and there might be a simple way to convert between them.

Impulse response & frequency response

Suppose we know the frequency response to a system and want to know what its output is to an impulse. Remember how to do that?

Output amplitude $f = \text{Input amplitude } f \times \text{Amplitude response } f$

And just for your information:
The same applies for the phase spectrum...

Output phase $f = \text{Input phase } f + \text{Phase response } f$

Calculating the spectrum of an impulse directly ...

- is really hard!
- So we’ll calculate the spectrum of a rectangular pulse ...
- and imagine it getting narrower and narrower.
- Start with a 2 ms pulse
Waveform and spectrum of a 2 ms pulse

Looking carefully at the spectrum

Easier to see spectrum on a dB scale

Amplitude spectrum of a 1 ms pulse
The spectrum of a rectangular pulse (zeros at multiples of 1/pulse duration)

- amplitude spectrum
  - flat (equal amplitude at all frequencies)
  - what other signal has this property?
  - so what else must be different?

- phase spectrum
  - 0 everywhere (when impulse is at time=0)

The spectrum of an impulse (infinitely narrow rectangular pulse)

- amplitude spectrum
  - flat (equal amplitude at all frequencies)
  - what other signal has this property?
  - so what else must be different?

- phase spectrum
  - 0 everywhere (when impulse is at time=0)

Impulse response & frequency response

Suppose we know the frequency response to a system and want to know what its output is to an impulse. Remember how to do that?

Output amplitude (f) = Input amplitude (f) \times \text{Amplitude response (f)}

And just for your information:
The same applies for the phase spectrum...

Output phase (f) = \text{Input phase (f)} + \text{Phase response (f)}

What is the amplitude spectrum of an impulse?

Answer: constant (k)

→ The amplitude spectrum of a system impulse response is simply the amplitude response of the system.
Response of an LTI system to an impulse

Input signal → SYSTEM → Output signal

0

phase spectrum

phase response

phase spectrum

amplitude spectrum

amplitude response

amplitude spectrum

IMPORTANT SUMMARY

The amplitude spectrum of the impulse response is simply the amplitude response of the system.

For your further information: The phase spectrum of the impulse response is simply the phase response of the system.

Other impulse responses