



Lecture 1-5: Frequency Response

Overview

1. **Signals and Systems:** we draw a distinction between **signals** as physical instances of waveforms, and **systems** as channels which may modify signals passing through them. Thus sound, vibrations, light, and electricity are signals; while telephones, amplifiers, rooms and vocal tracts are systems. Do not confuse methods and terminology for describing signals with methods and terminology for describing systems.
2. **Characterising Systems:** we are concerned with how to make a quantitative description of systems. We start by considering a simple resonator as an example of the simplest interesting system. We already know that the response of a simple resonator to an impulse can be characterised as a **damped sinusoidal waveform** where the frequency of the sinusoid is the system's natural frequency, and the amount of damping in the system affects how rapidly the amplitude diminishes with time (see figure 1-5.1). However this approach doesn't seem to extend simply to more complex systems which have more than one resonant frequency, so instead we consider how systems respond to sinusoidal inputs.
3. **Frequency Response:** we define the **response** of a system to a sinusoidal input as the ratio of the output amplitude to the input amplitude. Thus an amplifier which increases the size of signals by a factor of 10 will have a response of 10. Perfect amplifiers will amplify a sinusoidal signal by the same amount regardless of the frequency of the sinusoid. However in general a system may not have the same response value at all frequencies. A simple resonator, for example, prefers to vibrate at its natural frequency; so it will have a bigger response value at its natural frequency than at any other frequency. We can draw a graph of the response of a system to sinusoidal inputs as a function of the frequency of the input. This is called a **frequency response graph**, and we can use it to characterise the behaviour of *any* system.
4. **Output = Input x Response:** given the frequency response graph for a system we can predict how any signal will be affected by the system: we simply multiply the input spectrum by the frequency response to get the output spectrum (see figure 1-5.2). The reason this is possible is that any input signal can be considered to be a sum of sinusoids (by Fourier analysis) and the output from the system must be the sum of the outputs of the system to each input sinusoid in turn [this is a result of *linearity*: $f(a+b)=f(a)+f(b)$].
5. **Simple Resonator Response:** the frequency response of a simple resonator can be characterised by just two numbers: the **natural frequency** (or resonant frequency), and the **bandwidth**. The natural frequency is just the frequency at which the response peaks. The bandwidth is defined as the width of the peak at 3dB below the peak response value. Bandwidth is measured in hertz (Hz) and is the preferred means of measuring the degree of damping, since resonators with a lot of damping will have a wide bandwidth, while those with little damping will have a narrow bandwidth (see figure 1-5.1).

Readings

Choose at least one from:

- Ladefoged, Elements of Acoustic-Phonetics (2nd edition), Chapter 5: Resonance. *Gentle introduction to resonance.*
- Rosen & Howell, Signals and Systems for Speech and Hearing (1st edition), Chapter 6: The Frequency Response of Systems, pp70-75. *Essentials of frequency response.*

Learning Activities

You can help yourself understand and remember this week's teaching by doing the following activities before next week:

1. Using examples, write an explanation of the difference between a signal and a system.
2. List the measurable properties of signals that we have met and contrast them to the measurable properties of systems.
3. Explain in your own words what a frequency response graph measures and how it can be measured in the laboratory.
4. Draw a graph of the frequency response of a simple resonator, demonstrating the effect of a change in damping on its shape.

If you are unsure about any of these, make sure you ask questions in the lab or in tutorial.

Reflections

You can improve your learning by reflecting on your understanding. Here are some suggestions for questions related to this week's teaching.

1. What units are used to measure response?
2. How does the volume control on an amplifier affect its response?
3. Try and explain what a frequency response graph tells us about a system.
4. What would be the frequency response graph of an ideal amplifier? An ideal attenuator?
5. Think of ways of measuring the frequency response of some simple systems: for example: a hi-fi amplifier, a loudspeaker, or a room.
6. What is meant by a 'time domain' description of how a signal is affected by a system?
7. What is meant by a 'frequency domain' description of how a signal is affected by a system?
8. How do you measure bandwidth from a frequency response graph of a simple resonator?
9. What is wrong with the statement: "the resonator had a fundamental frequency of 100Hz"

Quiz

Which of these terms are applicable to signals and which to systems?:

1. Fundamental frequency
2. Natural frequency
3. Spectrum
4. Amplitude
5. Response
6. Damping

Figure 1-5.1 Effects of Damping on the Output of a Simple Resonator Excited by a Pulse

The figure shows how the amount of damping affects how a system responds in time to an impulse, and how the frequency response graph changes in bandwidth.

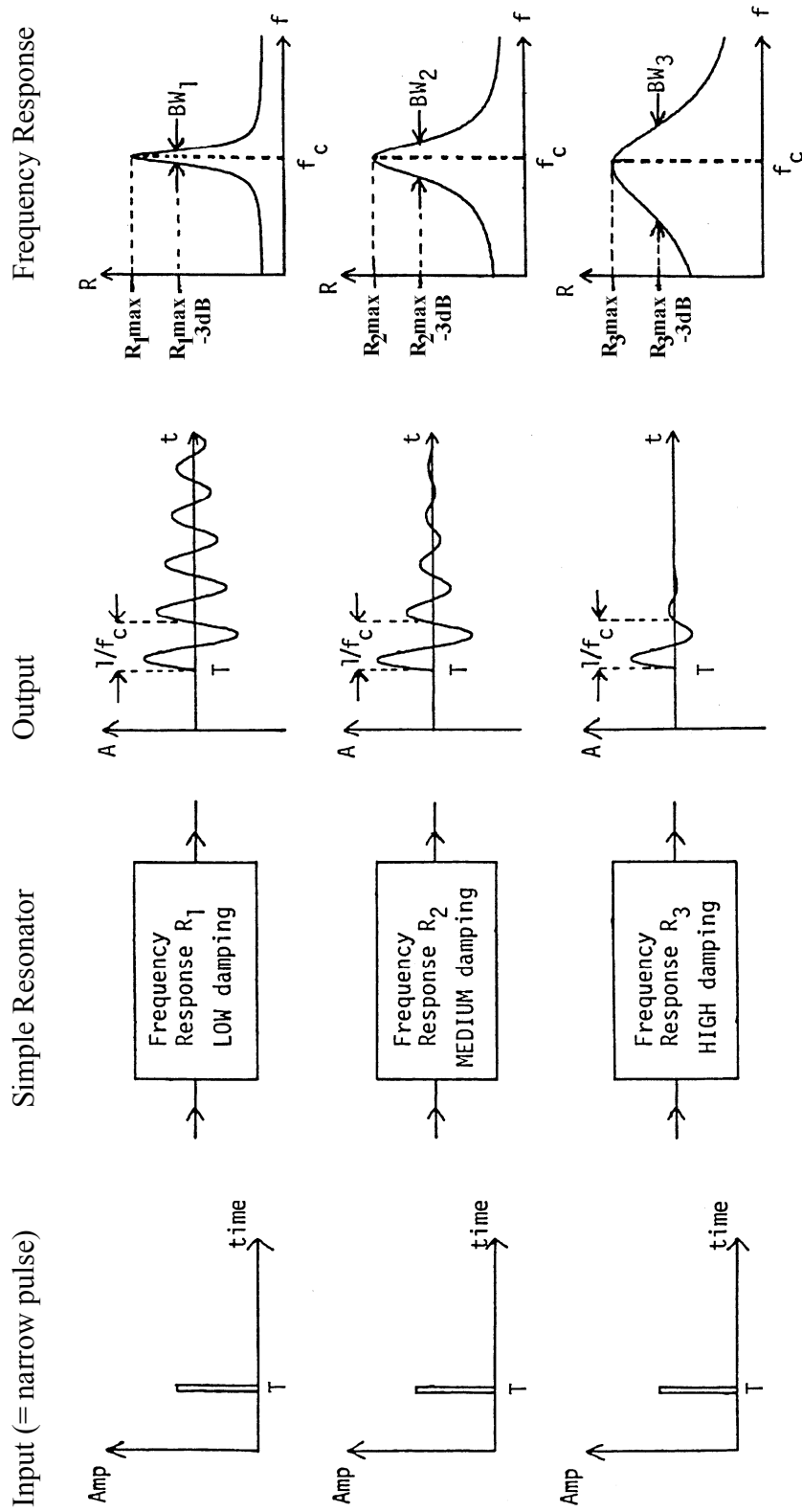
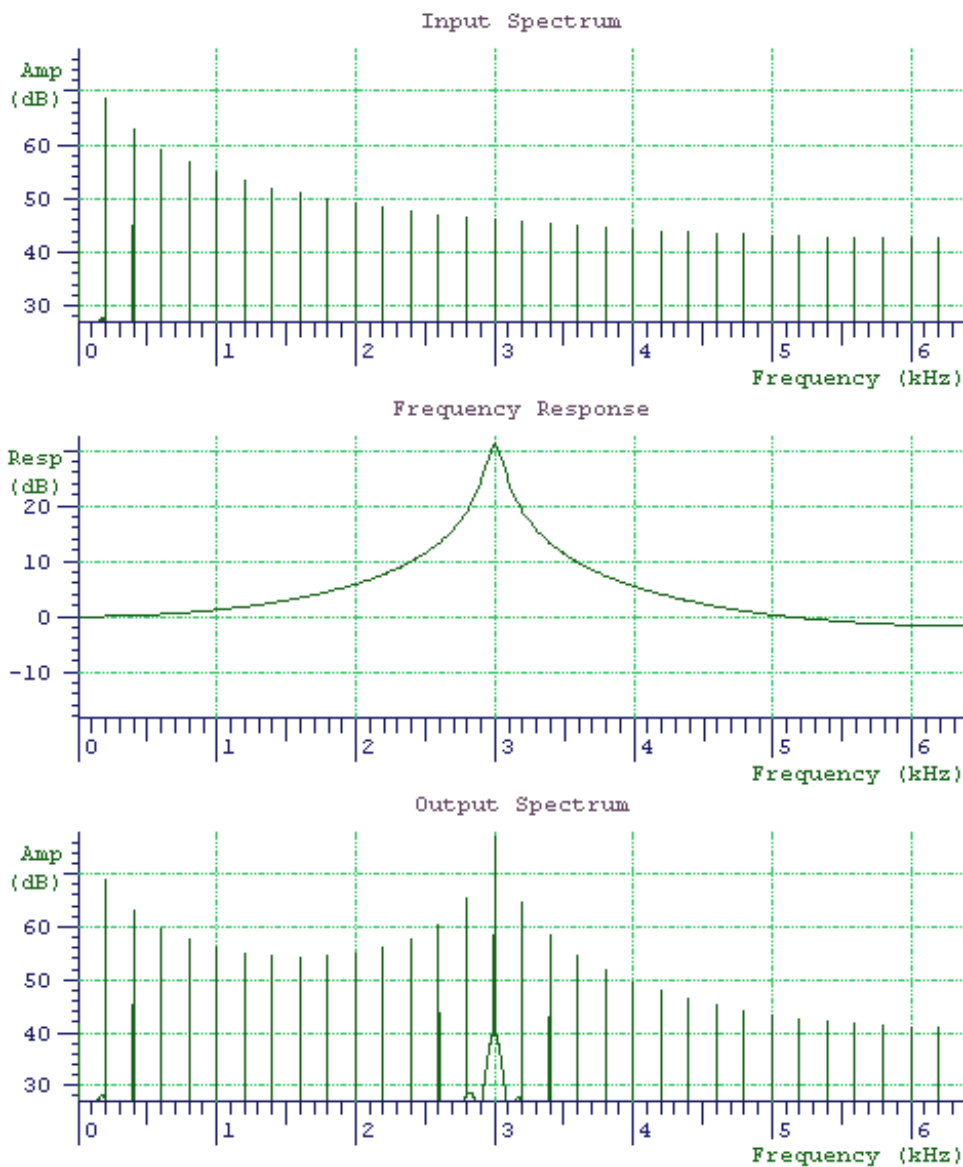


Figure 1-5.2 Output = Input x Response



Given the spectrum of an input signal and the frequency response of a system we can always predict the spectrum of the output signal by multiplying the spectrum by the response. In the figure above, the amplitudes are all encoded in decibels, so that this multiplication in amplitude is demonstrated through the addition of decibel values. Thus at 3000Hz, an input component at 47dB is amplified by +31dB to obtain its output size of 78dB.

Lab 1-5: Acoustic Resonator Frequency Response

Introduction

An air-filled cavity can act as a resonator. By this we mean there are preferred frequencies at which the air in the cavity likes to vibrate. The simple acoustic resonator (or Helmholtz resonator) comprises an enclosed volume of air and a small aperture. Such a resonator has a single natural frequency related to the volume of the air in the cavity and the dimensions of the aperture. In contrast, an acoustic resonator that is a simple tube open at one end (such as the vocal tract) has more complex behaviour in which the air can resonate simultaneously at a number of frequencies.

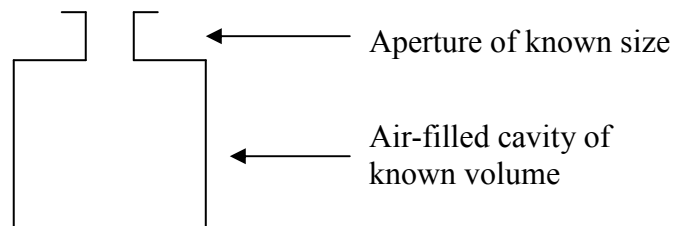


Figure 1. Helmholtz resonator

To characterise the frequency characteristics of any system, we plot its *frequency response* graph. This quantifies how the system changes sinusoidal signals passing through it as a function of the frequency of those sinusoids. The response at frequency f is simply the ratio of the output amplitude at frequency f to the input amplitude at frequency f .

Scientific Objectives

- To measure the frequency response of a Helmholtz resonator at two different sizes
- To measure the frequency response of a Helmholtz resonator with and without additional damping.

Learning Objectives

- To understand the concept of frequency response, and to know how to calculate a frequency response graph.
- To appreciate the connection between bandwidth and damping.
- To learn how to quantify the natural frequency and bandwidth of a simple resonator.

Apparatus

The acoustic resonator consists of a simple tube, closed at one end by movable piston and at the other by a disk with a small aperture (of fixed size). The volume of the resonator can be varied by moving the piston and is directly proportional to the distance between the top of the piston and the underside of the disk.

The column of air in the tube is set vibrating by a signal from a sinewave source played into a loudspeaker in the piston. The amount of vibration of the air column is recorded by a microphone in the piston. The input and the output amplitudes are displayed on two millivoltmeters.

You may like to make a sketch of the equipment and how it is connected.

Method

Condition 1. Set the tube length to about 9cm. Find the resonant frequency and adjust the oscillator amplitude so that you get an **output** amplitude reading just above 9mV on the voltmeter scale. Make measurements of the input and output voltmeter readings for frequencies in the region 250Hz to 450Hz. You'll need to make more detailed measurements near the peak than at the edges.

Condition 2. Set the tube length to about 6cm. Find the resonant frequency and adjust the oscillator amplitude so that you get an **output** amplitude reading just above 9mV on the voltmeter scale. Make measurements of the input and output voltmeter readings for frequencies in the region 250Hz to 450Hz. You'll need to make more detailed measurements near the peak than at the edges.

Condition 3. Keep the tube length set to about 6cm. Add a double layer of cloth over the air-hole and secure in place with a rubber band. Find the resonant frequency and adjust the oscillator amplitude so that you get an **output** amplitude reading just above 9mV on the voltmeter scale. Make measurements of the input and output voltmeter readings for frequencies in the region 250Hz to 450Hz. You'll need to make more detailed measurements near the peak than at the edges.

Observations

Set out the results in three tables, one for each condition, with the headings:

- a) Excitation frequency (Hz)
- b) Input amplitude (mV)
- c) Output amplitude (mV)
- d) Response (i.e. ratio of (c) to (b)).
- e) Response (dB) (i.e. $20\log_{10}(d)$).

Plot all three conditions on the **same** graph with a horizontal axis of Frequency (Hz) and a vertical axis of Response (dB). You can use a lab PC to plot your graphs if you prefer.

Concluding Remarks

1. Comment on the change in frequency response caused by the change in tube length.
2. Comment on the change in frequency response caused by the additional damping.
3. The **bandwidth** of a simple resonator is defined as the width of the frequency region within which the resonator response is within 3dB of the peak response. From your graphs, measure the bandwidth in the three conditions.
4. How might the effect of damping on bandwidth shown in this experiment be demonstrated using a pendulum instead of the acoustic tube?