Signals, systems, acoustics and the ear

Week 4

Signals through Systems
Crucial ideas

- *Any* signal can be constructed as a sum of sine waves
- In a *linear time-invariant* (LTI) system, the response to a sinusoid is the same whether it is on its own, or as one component of a complex signal
  - No interaction of components
- An LTI system *never* introduces frequency components not present in the input
  - A sinusoidal input gives a sinusoidal output of the same frequency
- Hence, the output is the *sum* of the individual sinusoidal responses to each individual sinusoidal component of the input
Fourier analysis

waveform $\rightarrow$ spectrum

Fourier analysis

LTI system (frequency response)

spectrum $\rightarrow$ waveform

Fourier synthesis
Six steps to determining system output to any particular input

1. Obtain the system’s amplitude response
2. Obtain the system’s phase response
3. Analyse the waveform to obtain its spectrum (amplitude and phase)
4. Calculate the output amplitude of each component sinusoid in the input spectrum
5. Calculate the output phase of each sinusoid
6. Sum the output component sinusoids
A particular example

input signal \rightarrow \text{system} \rightarrow \text{output signal} = ?
Step 1: Measure the system’s response

For example, by using sinewaves of different frequencies (as for the acoustic resonator)

Here the response has a gain of
1 for frequencies up to 250 Hz
0 for frequencies above 250 Hz

Assume phase response is a phase shift of zero degrees everywhere
Step 2: Sawtooth amplitude spectrum

\[ A(n) = \frac{A(1)}{n} \]

(A is the amplitude of a harmonic, index \( n \) is harmonic number)

\( A(1) \) is for this example 1 volt
Sawtooth phase spectrum

All components have a phase of $-90^\circ$ (relative to a cosine)
Remember!

- Response = Output amplitude/Input amplitude
- So on linear scales ...
  - Output amplitude = Response x Input amplitude
- But on dB (logarithmic) scales
  - Output amplitude = Response + Input amplitude
  - because log(a x b) = log(a) + log(b)
- For phase
  - Output phase = Response phase + Input phase
Response to harmonic 1 (100 Hz)

<table>
<thead>
<tr>
<th>Input amplitude</th>
<th>Response gain</th>
<th>Output amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 V</td>
<td>1</td>
<td>?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input phase</th>
<th>Phase shift of response</th>
<th>Output phase</th>
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<tbody>
<tr>
<td>-90</td>
<td>0</td>
<td>?</td>
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</table>

![Graph showing frequency response](image-url)
Graph of signal - system - output for harmonic 1

1 V

x 1

= 1 V
### Response to harmonic 2 (200 Hz)

<table>
<thead>
<tr>
<th>Input amplitude</th>
<th>Response gain</th>
<th>Output amplitude</th>
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</thead>
<tbody>
<tr>
<td>1/2 V</td>
<td>1</td>
<td>?</td>
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<th>Input phase</th>
<th>Phase shift of response</th>
<th>Output phase</th>
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<tbody>
<tr>
<td>-90</td>
<td>0</td>
<td>?</td>
</tr>
</tbody>
</table>

![Graph of Frequency vs. Response](image)
Graph of signal - system - output for harmonic 2

0.5 V \times 1 = 0.5 V
Response to harmonic 3 (300 Hz)

<table>
<thead>
<tr>
<th>Input amplitude</th>
<th>Response gain</th>
<th>Output amplitude</th>
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<tbody>
<tr>
<td>1/3 V</td>
<td>0</td>
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<td>-90</td>
<td>0</td>
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![Graph showing response to harmonic 3]
Response to whole signal
Waveform of output

100 Hz

200 Hz

+ sum
A realistic amplitude response

-3 dB point
A phase response
Sawtooth Wave:
Input - System - Output

Note that multiplications are done 'all at once'
Output waveform

realistic lowpass filter

for comparison: ideal lowpass filter
Linear vs. logarithmic amplitude scales

linear amplitude

logarithmic amplitude

multiplication

addition

input spectrum

frequency response

output spectrum
Linear vs. logarithmic *frequency* scales

Logarithmic amplitude scales matter for calculations. Logarithmic frequency scales are a matter of convenience.
Using an aperiodic input (white noise):
A *continuous* spectrum

Note that additions are done ‘all at once’
Consider this frequency response (what is it?)

- Input signal
- Input spectrum
- Frequency response
- Output signal
- Output spectrum
White Noise

input

output
Single pulse
input

output
More complex examples
Bandpass filters & filterbanks
Practical spectral analysis

- Most analogue signals of interest are not easily mathematically specified ...
  - so applying a Fourier transform directly (through an equation) is not possible

- Digital techniques allow the use of the FFT ...
  - simply by sampling the waveform values

- How was this done back in the day? Or even now, in analogue form?

- What kind of LTI system separates out frequency components?
Try this out on an old friend ...

Sawtooth amplitude spectrum

Time waveform

Amplitude spectrum

5 ms
Need a bandpass filter with variable centre frequency

\[
20 \log(gain) = -3 \quad \therefore \quad gain = 10^{\frac{-3}{20}} \approx 0.7
\]
Tune filter to 200 Hz
Tune filter to 300 Hz
Tune filter to intermediate frequencies
To construct the spectrum
Can do this in two ways ...

• As shown, with a tunable bandpass filter
  – cheap to implement, slow to run

• Or, with a *filter bank*
  – A set of bandpass filters whose centre frequencies are distributed over a desired frequency range
  – fast because of parallel processing but expensive in hardware

• Exotic fact you can ignore
  – an Fourier analysis can be thought of as implementing a filter bank
What filter properties affect the output of a filterbank?

- ????
- ????. of filters in a filter bank determines the resolution of the spectrum
- Need to space filters relative to ????
- Why?
  - don’t want holes in the spectrum
  - could miss spectral components
How the properties of a filter bank influence signals through it:
I. Resolution in frequency

Consider a signal that consists of two sinusoids reasonably close in frequency, which are to be analysed in a filter bank.
Filtering through narrow filters
Filtering through wide filters
A more extreme example
Narrow band filters

input wave

500 Hz + 580 Hz
Wide band filters

500 Hz filter output

input wave

500 Hz + 580 Hz

580 Hz filter output
Spectral analysis with a filter-bank:

No single unique spectrum!
Example filter bank and analysis (bandwidth ≈ 100 Hz)
Example filter bank and analysis (bandwidth ≈ 500 Hz)
Impulses through narrow and wide filters
Bandwidth & Damping

• Two ways of describing the same thing:
  – **Narrow** Bandwidth = **Low** Damping
  – **Wide** Bandwidth = **High** Damping
Summary

• Bandpass filters with a long impulse response have narrow frequency responses.
• Bandpass filters with a short impulse response have broad frequency responses.
How the properties of a filter bank influence signals through it:

II. Resolution in time

Consider a signal that consists of two impulses reasonably close in time, which are to be analysed in a filter bank.
Filtering through a wide filter
Filtering through a narrow filter
Summary

• Filter banks which consist of relatively narrow filters are good for seeing fine spectral detail ...
  – but poor for temporal detail

• Filter banks which consist of relatively wide filters are good for seeing fine temporal detail ...
  – but poor for spectral detail
Applying these concepts to a complex periodic wave consisting of 20 equal-amplitude harmonics of 100 Hz
A complex periodic wave consisting of 20 equal-amplitude harmonics of 100 Hz
Narrow-band (50 Hz) filtering at 200, 250, 300, 350 and 400 Hz

what do you see?
Wide-band (300 Hz) filtering at 200, 250, 300, 350 and 400 Hz

what do you see?
What does a filter bank do to a speech waveform?

a 6-channel filter bank
Narrow bands of speech at different frequencies: Individual outputs from a filter bank

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Of course, you need many more filters in the filter bank than seven.
What can you use filter banks for?

Other than spectral analyses ...
To make spectrograms or voiceprints ...
To make a graphic equaliser ...
To process sounds for a multi-channel cochlear implant (an electronic filter bank substitutes for the basilar membrane).
In hearing aids ... 

Shape the spectrum of incoming sounds to compensate for the hearing loss frequency regions with bigger loss get greater gain 

a graphic equaliser!
In computational models of the auditory periphery.

Imagine that each afferent auditory nerve fibre has a bandpass filter attached to its input.