Please answer all of the following questions.

1) Consider the amplitude responses of the following two systems.
   a) What kind of a filter does each represent?
   b) Draw the amplitude response of System 2 on linear gain and linear frequency scales.
   c) Now draw the amplitude response of a cascade of these two systems, System 1 followed by System 2, on dB and linear frequency scales.
   d) What would be the amplitude response of the cascade if the position of the two systems were reversed? Explain your reasoning.
   
   (10 points total)
Scoring Q1

a) 2 points: Band pass, high pass.

b) 3 points: Linear gain of system 2

c) 3 points: frequency response of cascade

d) 2 points: It doesn’t matter if the order of the systems is reversed because the gain values are added and addition doesn’t depend on the order.

![Linear gain for System 2](image1)

![Total response](image2)
2) Draw input and output spectra of the following 4 signals passed through System 1 in question 5, over the frequency range 0-2 kHz, on dB and linear frequency scales. Ensure that your labels are accurate, and that your amplitude scales are consistent across the input and output graphs. **(10 points)**

a) A sinusoid at 300 Hz
b) White noise
c) An impulse
d) A periodic train of impulses with a period of 0.01 s

**Scoring Q2**

a) sinusoid at 400 Hz: Input spectrum must be a single component; output spectrum identical to frequency response of 1st system. Must be labeled dB SPL or some other appropriate measure.

b) White noise: Input spectrum must be a continuous flat line; output spectrum identical to frequency response of 1st system. Must be labeled dB SPL or some other appropriate measure.

c) Impulse: Input spectrum must be a continuous flat line; output spectrum identical to frequency response of 1st system. Must be labeled dB SPL or some other appropriate measure.

d) Impulse train: Input spectrum must have harmonics of equal size at 100 Hz; output spectrum also with harmonics but with spectral envelope as in System 1. Must be labeled dB SPL or some other appropriate measure.

Sinusoid: 1 point total

All other waves: 1 point/input spectrum; 2 points/output spectrum
3) Draw two cycles of a digital 50 Hz sinusoid at a peak voltage of 20 µV, sampled at 400 times per second. What processes are necessary to convert an analogue to a digital waveform? What are the limitations of these processes? (15 points total)

**Scoring Q3**

![Graph](image)

Graph worth 5 points

A-toD conversion requires:

5 points: Quantisation to a given number of bits; always better to have more but a good enough representation can be obtained from 16 bits or so

5 points: Sampling at discrete points in time; Must be at twice the frequency present in the signal; signals must be low-pass filtered (anti-aliasing filters) if this is not true.
4) Consider a periodic train of very narrow pulses at 100 Hz, whose fundamental component is at a level of 10 dB re 1 mV. This signal passes through a high-pass filter which has a passband gain of -10 dB at 400 Hz and upwards, and a low frequency slope which rolls off at 12 dB/octave from there.

a) Draw the amplitude spectrum of the input wave on a linear amplitude scale over the frequency range 0 – 1000 Hz (use a linear frequency scale).

b) Draw the frequency response of the system using dB gain and log frequency over the range 100 – 1000 Hz.

c) Using dB amplitude scales, draw the amplitude spectrum of the input wave, and the output of this system to it, over the frequency range 100 – 1000 Hz.

d) Would the input waveform be changed after passing through the system? Give reasons for your answer but you need not draw the output wave.

(20 points total)

Scoring Q4

a) (level=3.16 mV) 4 points

c) On dB scales, looks exactly the same but at a level of 10 dB re 1 mV: 1 point

d) output wave would have to be different because its spectral composition is different = 5 points
Calculations for problem above

<table>
<thead>
<tr>
<th>harmonic frequency</th>
<th>frequency level (Pa)</th>
<th>dB re 1 mV</th>
<th>frequency response</th>
<th>gain (dB)</th>
<th>output levels</th>
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5) It is often said that the function of the basilar membrane can be likened to that of a filter bank. Describe what a filter bank is, and how the notion of a filter bank can be used to understand cochlear function and the relevant aspects of auditory nerve firing patterns. What properties would the filter bank need to have in order to best mimic the functioning of the inner ear? (20 points)

Scoring Q5

(2 points/idea below + 4 for extras/quality/coherence)

- A filter bank is a collection or set of band-pass filters
- whose centre frequency varies over some range
- As the basilar membrane (BM) does a kind of frequency analysis, its function can be compared to a filter bank
- each auditory nerve fibre responding to the acoustic world through a single bandpass filter.
- Filterbank should span 20 Hz-20 kHz
- have bandwidths increasing with increasing frequency)
- spaced on a quasi-logarithmic scale.
- be nonlinear (increase bandwidth and decrease gain with increasing level)
6) Almost all systems clip signals that are too large to be handled by them, so any system can only be considered to be linear time-invariant (LTI) over some limited range of levels. Suppose you had a system that acted as a perfect amplifier with a gain of 4. However, the magnitude of the output voltage is strictly limited to 10 V peak-to-peak (so the minimum voltage is -5 V and the maximum 5 V). *(25 points)*

a) Draw the frequency response of the system, on dB and linear frequency scales (0-3 kHz), assuming the input voltage to be 1 V.

b) Draw output waveforms for 2 cycles of a 400 Hz sinusoid when: (i) the peak voltage of the input is 1 V, and (ii) the peak voltage of the input is 2 V. In terms of a general description, what kinds of output waves do you obtain in the two cases (aperiodic, simple, complex, periodic)?

c) Make your best guess as to what the spectrum of the output to a 2 V input sinusoid at 400 Hz would look like, and draw it (using linear scales on both axes).

d) Draw the input/output function of the system for a 400 Hz sinusoid for peak voltages ranging from 0 V to 3 V.

e) Is the system homogeneous? Why or why not?

f) Is the system time-invariant? Why or why not?

g) From what you have shown above, what are the two ways you can show that this system is not LTI for input voltages ranging from 0-3 V?

**Scoring Q6**

![Frequency Response Graph]

*a) 2 points*
b) 2 points: sinewave output: simple periodic wave
4 points: clipped sinewave: complex periodic wave
(1 point for classification of wave; residual points for graph)

c) 5 points: spectrum of clipped sine wave can only have harmonics at multiples of 400 Hz, with decreasing amplitude; appropriately labeled axes. +2 points for noting that only odd harmonics would be present.

d) 3 points for I/O function

e) 2 points: Homogeneous? no; as input/output function shows

f) 3 points: Time-invariant? yes; mention effect of delay

g) 4 points: not LTI: An input sinusoid results in an output that is not a sinusoid, as seen above right. And that the input/output function is not a straight line through the origin.