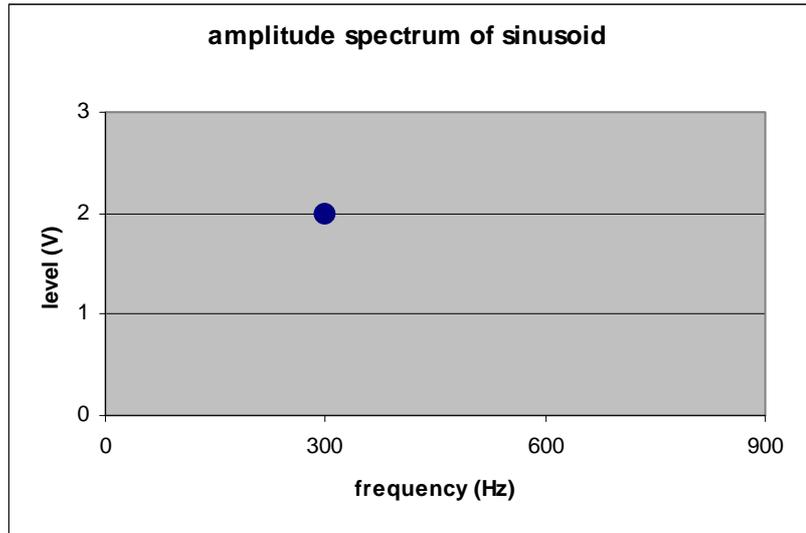


Answers to Exercises

Chapter 7, exercise 1:

A sinewave has, of course, a single spectral component:

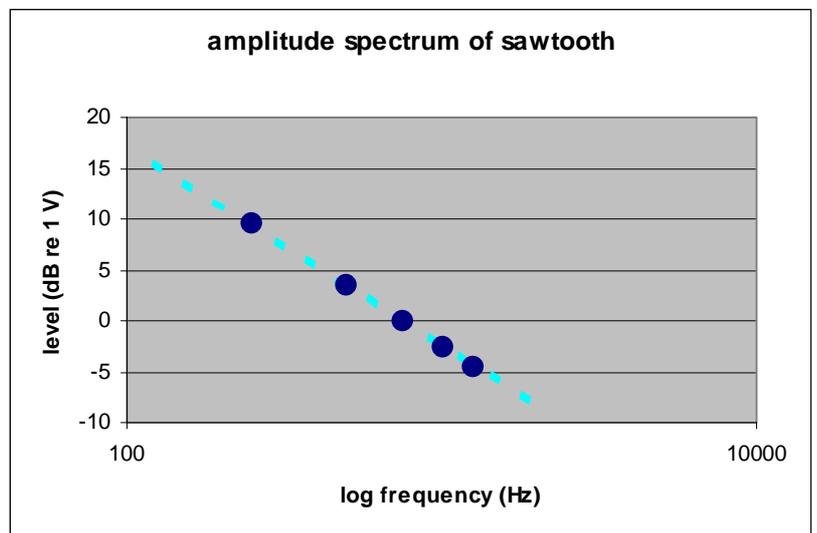
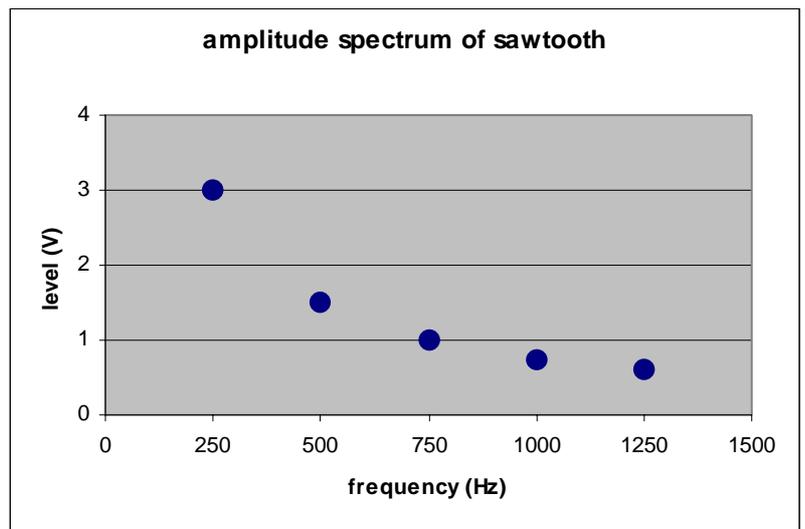


Chapter 7, exercise 3:

Note first that a sawtooth is a periodic waveform, hence all its spectral components will be at multiples of the fundamental. Since its period is 4 ms, it must have a fundamental frequency of $1/0.004 = 250$ Hz. Therefore, there will be harmonics at 250, 500, 750, 1000 and 1250 Hz. The amplitude of each harmonic is given by L/n where L is the level of the fundamental and n is the harmonic number. (Up until now we have concentrated on cases where the level of the fundamental was 1, so that the amplitude of each harmonic was simply $1/n$). Here $L = 3$ V, so the corresponding level of the harmonics is 3, 1.5, 1, 0.75 and 0.6 V. Once the level of each harmonic is calculated, the conversion to dB re 1 V is straightforward:

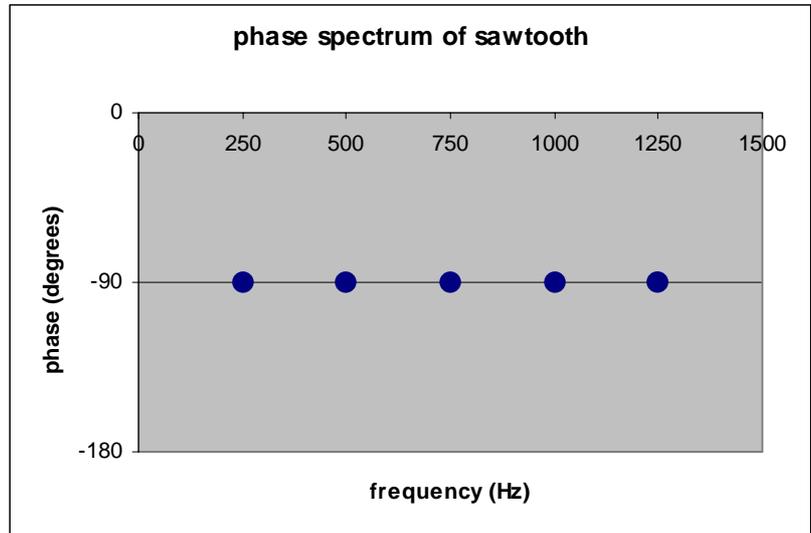
$\text{dB re 1 V} = 20 \log (x \text{ V}/1 \text{ V})$
 where x is level in V to be converted.

When the amplitude spectrum is plotted on dB and log frequency scales, you can see that the spectral envelope is a straight line of -6 dB/octave. You can calculate this by



seeing how the level of the harmonics change as frequency increases by each octave, or doubling of frequency. So the value at 250 Hz is 9.5 dB re 1 V (hereinafter written as dBV), while at 500 Hz, the level is 3.5 dBV, so since 500 Hz is one octave above 250 Hz ($500 = 2 \times 250$), the slope of the line is -6 (9.5-3.5) dB/octave. You can also get this number by looking at the change in level from 500 Hz to 1000 Hz (-2.5 dBV).

Having all the components in sine phase leads to a sawtooth, so the phase spectrum of the sawtooth looks like this →



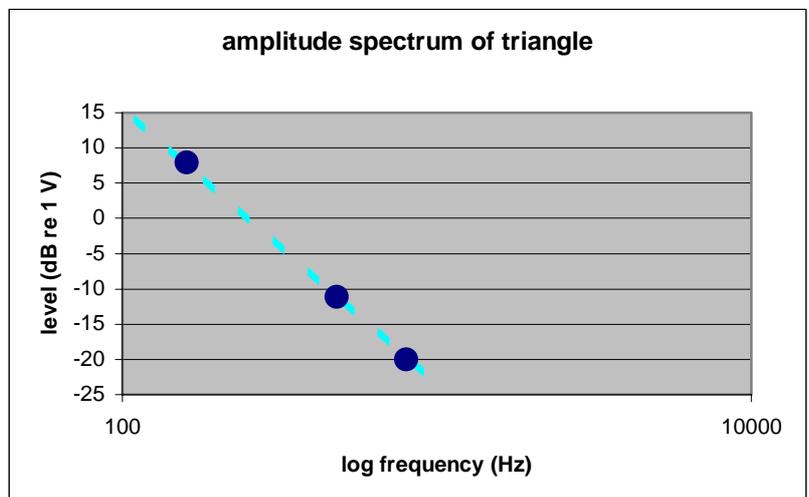
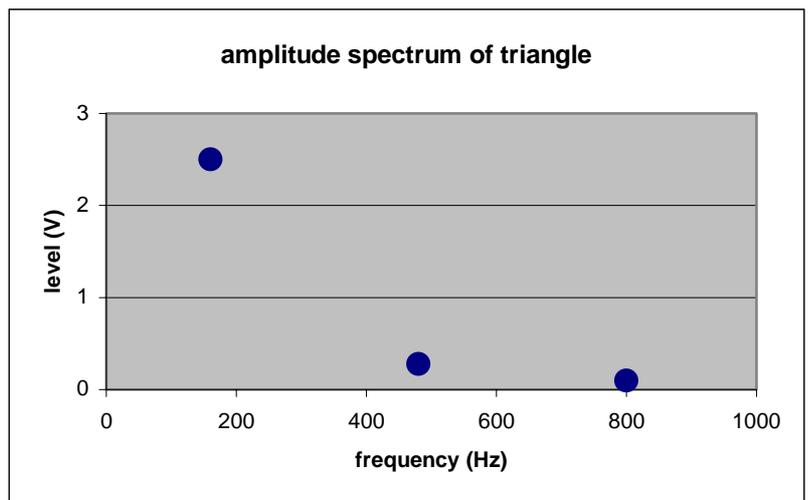
Chapter 7, exercise 4:

Note first that a triangle wave is a periodic waveform, hence all its spectral components will be at multiples of the fundamental. Unlike the sawtooth, it only consists of odd harmonics. Therefore, over the frequency range 0-800 Hz, there will be harmonics at 160, 480 and 800 Hz. The amplitude of each harmonic is given by L/n^2 where L is the level of the fundamental and n is the harmonic number. Here $L = 2.5$ V, so the corresponding level of the harmonics is 2.5, 0.28 and 0.1 V. Once the level of each harmonic is calculated, the conversion to dB re 1 V (dBV) is straightforward:

$$\text{dBV} = 20 \log (x \text{ V}/1 \text{ V})$$

where x is level in V to be converted.

When the amplitude spectrum is plotted on dB and log frequency scales, you can see that the spectral envelope is a straight line. Calculating its slope might seem a little tricky because it only has odd harmonics, but you can use the same formula to



calculate what the level of second harmonic would be (one octave above the fundamental) if it existed. Calculating $L/n^2 = 2.5/2^2 = 0.63 \text{ V}$, you then need to convert this to dB, resulting in -4.08 dBV , so since $2.5 \text{ V} = 7.96 \text{ dBV}$, this is a slope of just about -12 dB/octave ($-4.08 - 7.96 = -12.04$).

For those of you willing to calculate logarithms to the base 2, you could convert the frequency ratio of any two spectral components into octaves, and divide this into the number of dB between those two frequencies. In particular, if f_1 and f_2 are the frequencies of two spectral components which are at levels of L_1 and L_2 , then the spectral slope between them is:

$$(L_2 - L_1) / \log_2(f_2 / f_1) .$$

For the particular case here of the harmonics at 160 and 480 Hz:

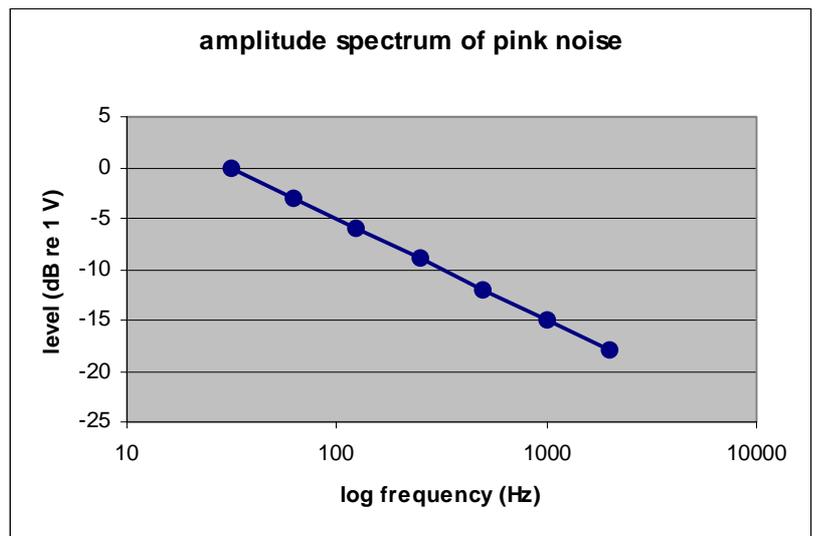
$$(-11.13 - 7.96) / \log_2(480/160) = -19.08 / \log_2(3) = -19.08 / 1.58 = -12.04$$

(Note that Excel will do all these calculations readily as there is a log function that can operate with any value for the base.)

All calculations can be found in the Excel file.

Chapter 7, exercise 7:

Excel will only draw ticks at multiples of 10 for log frequency, but it would be a lot easier to draw the ticks at octaves (31.25, 62.5, 125, 250, 500, 1000 and 2000 Hz). The dB reference is arbitrary, as long as the level decreases by 3 dB for each octave. A point cannot be drawn at 0 Hz because that is at $-\infty$. This also implies that the level of the noise would be infinitely high at 0 Hz, because the spectrum increases by 3 dB for every octave lowering (dividing frequency by 2), and you can never actually reach 0 by continually dividing by 2.



Chapter 7, exercise 10:

Because there is 100 Hz between the two frequencies of the spectral components at 1100 and 1200 Hz and each is a multiple of 100 Hz, the fundamental frequency of this signal is 100 Hz. Its fundamental period would be $1/100 = 0.01 \text{ s} = 10 \text{ ms}$.