## AUDLS001 Signals and Systems for Hearing and Speech:

## Final exam questions \& answers

1) Take two sinusoids of 2 kHz , both with a phase of $0^{\circ}$. One has a peak level of 1.2 Pa while the other has a peak level of 0.7 Pa . Draw the spectrum of the larger sinusoid (on linear scales) and 3 cycles of its waveform. Write down the formula for converting a measurement in Pa into dB SPL. What are the individual peak levels of the two sinusoids in dB SPL? What would be their peak level (in dB SPL) if added? What if one had a phase of $180^{\circ}$ while the other remained at $0^{\circ}$ ? What level would the sum of these two sinusoids be, in Pa and dB SPL? (10 points)

$$
d B S P L=20 \log (x P a / 20 \mu P a)
$$




|  | Pa | dB SPL |  |
| :--- | :---: | :---: | :---: |
| larger |  | 1.2 | 95.6 |
| smaller |  | 0.7 | 90.9 |
| sum (in phase) |  | 1.9 | 99.6 |
| sum (out of phase) |  | 0.5 | 88.0 |

2) A complex periodic waveform is made up from the following spectral components. For each combination, calculate the fundamental frequency and fundamental period. ( 10 points)

|  |  |  | Answer: <br> frequency (Hz) | Answer: <br> period (ms) |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| a) |  |  |  |  |  |
| frequencies | 50 | 100 | 150 | 50 | 20.00 |
| b) periods | 1 | 1.25 | 5 | 200 | 5.00 |
| c) frequencies | 2100 | 2240 | 2590 | 70 | 14.29 |
| d) periods | 2 | 1 | 0.5 | 500 | 2.00 |
| e) frequencies | 2000 | 2700 | 3000 | 100 | 10.00 |

3) Consider a wave which consists of the first 8 harmonics of a sawtooth wave whose fundamental period is 5 ms , and whose fundamental component has a level of 1 Pa . ( 20 points)
a) Draw its spectrum (on dB SPL and logarithmic frequency scales using the logarithmic graph paper supplied).
b) This wave is then put through a 'System X ' which results in the output wave having a spectrum in which all components are of equal amplitude at 0.2 Pa . Draw this spectrum on dB SPL and logarithmic frequency scales over the frequency range 100 Hz to 3.2 kHz .
c) Over the same frequency range, and again using dB and logarithmic frequency scales, draw the amplitude response of 'System X'.



Note: x-axis to go from $100 \mathrm{~Hz}, 200 \mathrm{~Hz}, 400 \mathrm{~Hz}, 800 \mathrm{~Hz}, 1.6 \mathrm{kHz}, 3.2 \mathrm{kHz}$. Section b) essentially the same graph as Section a) but all points at the same level of 80 dB SPL.
4) A particular telephone system has as its first stage a band-pass filter with cut-off frequencies of 300 and 3500 Hz , extremely steep roll-offs, and attenuation of the whole pass-band by -20 dB . Additionally, and after this stage, it produces harmonic distortion by adding three harmonics at 2,3 and 4 times the primary frequency component at $-25 \mathrm{~dB},-30 \mathrm{~dB}$, and -40 dB with respect to the level of the primary. First, draw a frequency response of this telephone band-pass filter. Now imagine passing a single sinusoid of 75 dB SPL though this system. Draw input and output spectra assuming the frequency of the sinusoid is 1 kHz . Draw another pair of input and output spectra assuming the frequency of the sinusoid is 5 kHz . ( $\mathbf{2 0}$ points)

Answer must include:
a) frequency response graph with a pass-band between 300 and 3500 Hz at -20 dB gain, appropriately labeled.
b) 4 amplitude spectra:

1) input 1 kHz : single spectral component at 75 dB SPL
2) output for 1 kHz : Components at:

| Frequency $(\mathrm{kHz})$ | Level (dB SPL) |
| :--- | :--- |
| 1 | $75-20=55$ |
| 2 | $55-25=30$ |
| 3 | $55-30=25$ |
| 4 kHz | Outside of <br> passband |

3) input 5 kHz : single spectral component at 75 dB SPL
4) an empty spectrum for the 5 kHz input wave (all frequencies are outside the pass-band)
5) It is often said that the function of the basilar membrane can be likened to that of a filter bank. Describe what a filter bank is, and how the concept of one can illuminate peripheral auditory function. What properties would the filter bank need to have in order to best mimic the functioning of the inner ear. ( $\mathbf{1 5}$ points)

A filter bank is a collection or set of band-pass filters, whose centre frequency varies over some range. As the basilar membrane (BM) does a kind of frequency analysis, its function can be compared to a filter bank. Imagine each auditory nerve fibre responding to the acoustic world through a single bandpass filter. Filterbank should span $20 \mathrm{~Hz}-20 \mathrm{kHz}$, have bandwidths increasing with increasing frequency, and spaced on a quasi-logarithmic scale.
6) Draw the waveform and spectrum of an impulse. Draw frequency responses for two simple resonators, both tuned to 2 kHz , but with one having a bandwidth of 50 Hz , and one with a bandwidth of 500 Hz . Draw the output waveforms and spectra if an impulse had been fed to each of these resonators. ( 20 points)
Waveform of impulse must be very narrow pulse, and spectrum flat across frequency, with appropriate axes. Frequency responses must look like sensible bandpass filters, with cutoff frequencies indicated. Spectra of output waves should be identical to resonator frequency responses. Time waveforms need to show some ringing, with more ringing for narrow bandwidth resonator. Extra points if ringing is at 2 kHz , and for especially accurate representations.
7) Consider a cascade of three systems with the following amplitude responses. Draw the amplitude response of the complete cascade. What would be the amplitude response of the cascade if the position of Systems 2 and 3 were reversed? ( $\mathbf{1 5}$ points)




It doesn't matter if the order of the systems is reversed.
total

8) Draw the input and output spectra of white noise passed through the first system in the cascade of question 8. (10 points)

Input spectrum must be a continuous flat line; output spectrum identical to frequency response of $1^{\text {st }}$ system. Must be labeled $d B$ or some appropriate linear measure by frequency.
9) Suppose you had a system that acted as a kind of rectifier by squaring the input. Here are sample input and output waveforms for a sinusoid of peak amplitude 1 V and frequency of 200 Hz :



What's the simplest way of knowing that this is not an LTI system? Is this system homogeneous? Additive? Time-invariant? Give reasons for your answers. (20 points)

LTI system must be linear (=additivity \& homogeneity) and time invariant. Explain.
An input sinusoid resulted in a non-sinusoidal output.
Homogeneous: no
Additive: no
Time-invariant: yes
10) Describe the processes that are necessary for a sound card in a computer to convert an analogue signal into a digital one, and then back again. Use graphs to make your points clearer. Make sure you mention and discuss, among others, the following terms: anti-aliasing filtering, the Nyquist rate, sampling, quantisation. Write down in a table the digital signal you would obtain if you sampled a single cycle of a 1 -volt 100 Hz sinusoid at 400 samples/second. ( $\mathbf{2 0}$ points)

Analogue-to-digital conversion involves:
Low-pass anti-aliasing filtering as only frequencies less than half the sampling frequency can be represented accurately. Higher frequencies than this are aliased into lower frequencies. The Nyquist rate is twice the highest frequency in the signal, and is the minimum rate needed for accurate representation.

Quantisation also necessary to store a number in a fixed number of bits. Essentially rounding. Digital-to-Analogue conversion involves:

Converting digital values into a voltage, resulting in a wave made of steps. Low-pass filtering to lose the sharp edges.

| time (ms) | time (s) |  | amplitude |
| ---: | ---: | ---: | ---: |
| 0 | 0 | 0 |  |
| 2.5 | 0.0025 | 1 |  |
| 5 | 0.005 | $2.65359 \mathrm{E}-06$ |  |
| 7.5 | 0.0075 | -1 |  |
| 10 | 0.01 | $-5.30718 \mathrm{E}-06$ |  |



