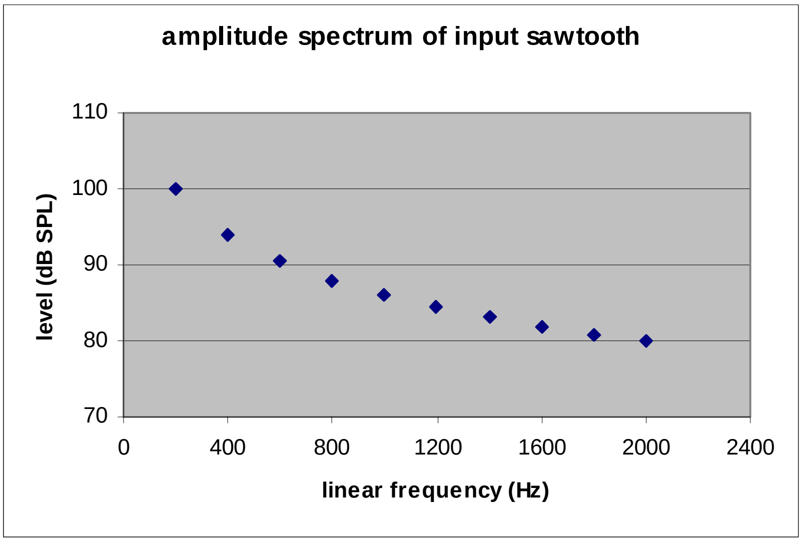
Answers to Exercises

Chapter 8, exercise 1: Do this only for a linear frequency scale

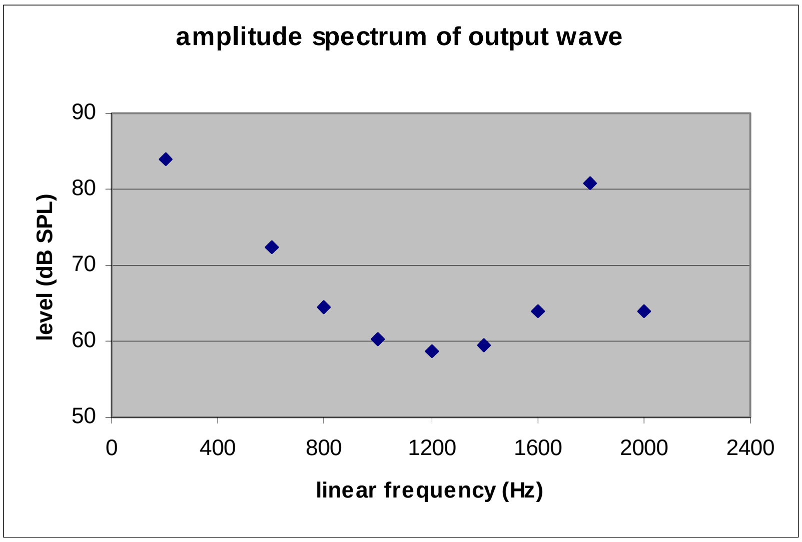
Note first that a sawtooth is a periodic waveform, hence all its spectral components will be at multiples of the fundamental (here, 200 Hz). The amplitude of each harmonic is given by *L/n* where *L* is the level of the fundamental and *n* is the harmonic number. (Up until now we have concentrated on cases where the level of the fundamental was 1, so that the amplitude of each harmonic was simply *1/n*). Once the level of each harmonic is calculated, the conversion to dB SPL is straightforward:

dB SPL = 20 log (*x* Pa/20 μPa) where *x* is level in Pa to be converted into dB SPL.

So the spectrum of the sawtooth looks like this →



The values of the frequency response must be read from the figure, but you only need values for the frequencies of the harmonics in the input spectrum. As both the spectrum of the sawtooth and the gain of the system are expressed in dB, add together these two values at each frequency to obtain the output spectrum here →



All calculations can be found in the Excel file.

Chapter 8, exercise 2

The input spectrum for a sinusoid has a single spectral component. The output wave is periodic, but is not a sinusoid, hence must have at least one harmonic. This is an example of *harmonic distortion*. A system which clips cannot be LTI because a sinusoidal input leads to a non-sinusoidal output.

Chapter 8, exercise 3

The values you needed to fill in are indicated in bold italics. Because this is a single LTI system, once you know the gain at a particular frequency, you can fill in that value for every other measurement at that frequency, for example, at ***(a)*** and ***(b)***. If you are given the input and output voltages (as at ***(c)***, you can work from the definition of gain:

$$\text{gain (dB)} = 20 \cdot \log(\text{Output/Input})$$

This formula can be re-arranged, by dividing both sides first by 20, and then raising 10 to the power given by each side. Recall that 10^x is the inverse function of log(x), and v.v., which is to say, one undoes the other. Once these two manipulations are done, you are left with:

$$10^{\text{gain}/20} = \text{Output/Input} = R$$

In order to calculate the Input voltage given the Output, or the reverse, you can use:

$$\text{Output} = \text{Input} \cdot 10^{\text{gain}/20}$$

$$\text{Input} = \text{Output}/10^{\text{gain}/20}$$

You should be able to see why this works. All we have done is convert the gain in dB into R, a linear value. And since:

$$R = \text{Output/Input}$$

You also know that:

$$\text{Output} = \text{Input}/R$$

$$\text{Input} = \text{Output}/R$$

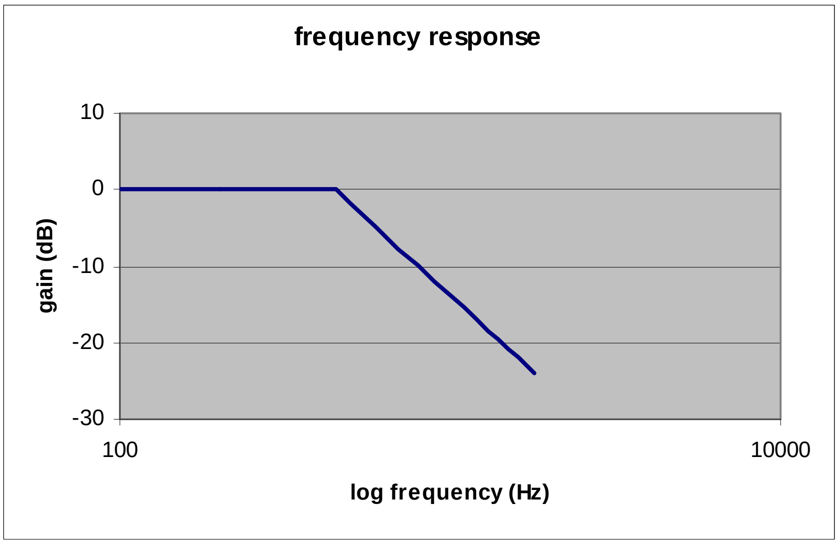
You really only need your calculator for the last line. You should remember that 6 dB corresponds to a factor of 2, and 20 dB to a factor of 10.

Frequency (Hz)	Volts input (V)	Gain (dB)	Volts output (V)
100	2	0	2
200	2	0	***2***
200	1	***0 (a)***	***1***
200	***4***	***0 (b)***	4
400	2	***-6 (c)***	1
400	***4***	***-6***	2
800	2	20	***20***
800	***0.4***	***20***	4
900	***3.16***	-10	1

Chapter 8, exercise 4: Use dB scales everywhere

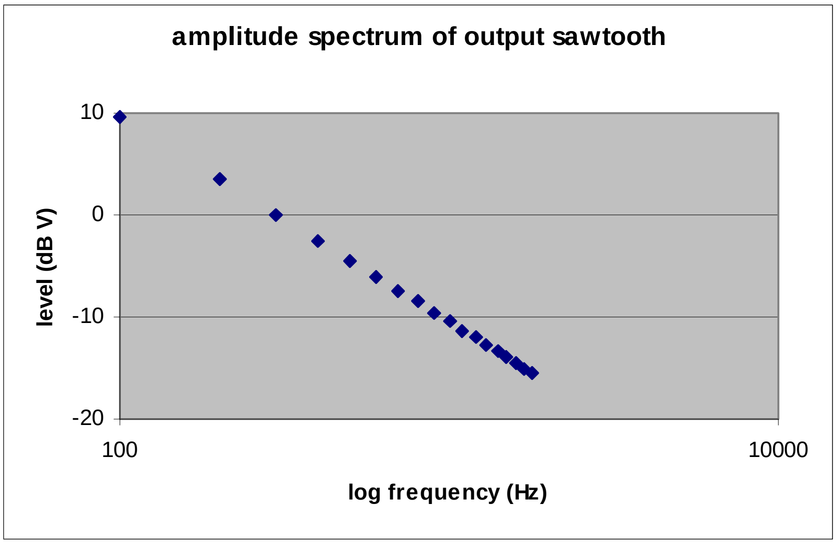
First, the low-pass filter will look like this →

This is easiest to draw on a frequency scale marked in octaves (which Excel will not do!), because you should realise that the gain would be 0 dB at 450 Hz, -12 dB at 900 Hz and –24 dB at 1800 Hz (all on a straight line).

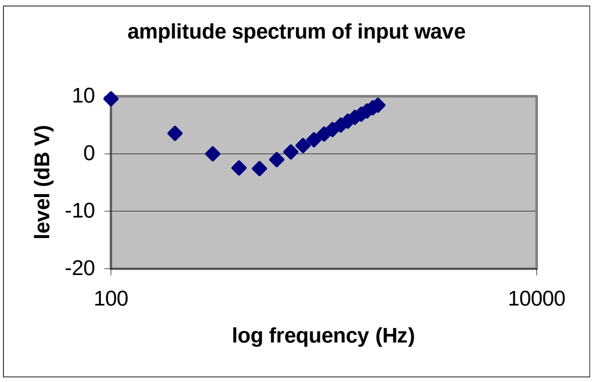


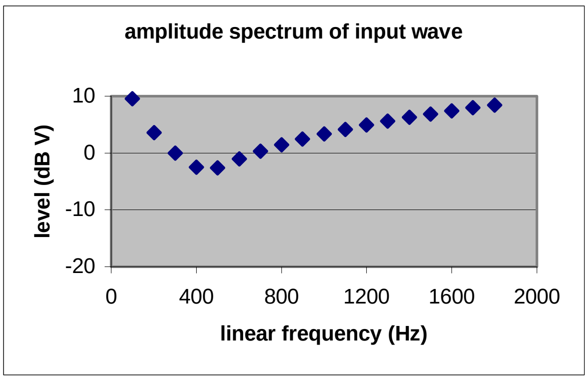
Calculating the spectrum of the output wave is very similar to the calculating the input wave for exercise 8.1 →

Finally, working from the fact that Output = Gain + Input (when all; values are expressed in dB), you can re-arrange this equation to give: Input = Output – Gain



Values of the gain can be calculated explicitly if you are a little clever mathematically, but are easiest to read off from a sketch like the one first given above. Subtracting the Gain values from the Output levels results in:



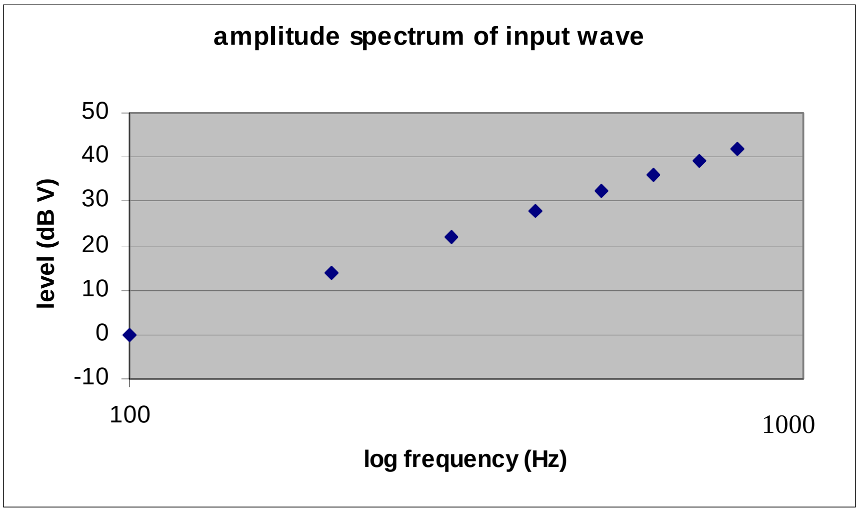


Again, all calculations can be found in the Excel file.

Chapter 8, exercise 6: Use dB scales everywhere, and it may be easier for you to assume that the level of the fundamental component in both input and output signals is 1 V.

Again, because the properties of the input waveform are specified in octaves, it will be easiest to work on a logarithmic frequency scale.

You should realise that the level of the spectral components in the periodic complex will be 0 dB re 1 V at 100 Hz, 14 dB re 1V at 200 Hz, 28 dB re 1 V at 400 Hz and 42 dB re 1 V at 800 Hz, and be linear on dB vs log frequency scales. (Note that the absolute level of the input signal is not given in the original question. In fact the shape of the frequency response does not depend on the overall level of the input or output but its vertical position will.)



Because you also know that the output signal has the same harmonics, but all at equal levels (we'll assume at 0 dB re 1 V), the frequency response of the system can be calculated by subtracting the levels of the input harmonics from the levels of the output harmonics. Because the input signal has a rising spectrum, and the output a flat spectrum, it should be clear to you that the system is a kind of low-pass filter.