Answers to Exercises

Chapter 4, exercise 2:

Homogeneity of this system is easily shown by plotting an input/output function, which would simply result in a straight line going through the origin, with a slope of 5.

For additivity, we need to show that:

If $inp_1(t) \rightarrow outp_1(t)$ and

$inp_2(t) \rightarrow outp_2(t)$

Then $inp_1(t) + inp_2(t) \rightarrow outp_1(t) + outp_2(t)$

For the particular case of this amplifier:

$inp_1(t) \rightarrow 5 \times inp_1(t)$

$inp_2(t) \rightarrow 5 \times inp_2(t)$

so that $inp_1(t) + inp_2(t) \rightarrow = 5 \times (inp_1(t) + inp_2(t)) = 5 \times inp_1(t) + 5 \times inp_2(t) = outp_1(t) + outp2(t)$

This follows from the property that *multiplication distributes over addition*, which is just an exotic way of saying that:

$$k \times (a + b) = k \times a + k \times b.$$

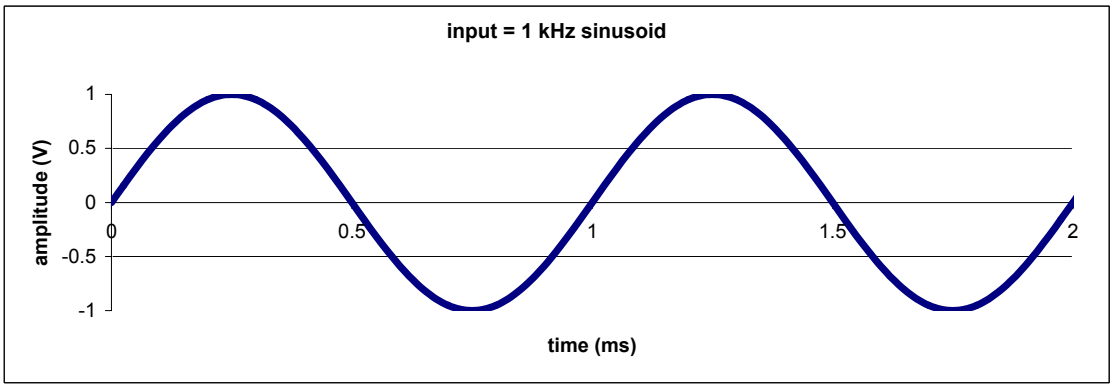
It is also time-invariant as no property of the system is changing over time, so there is no dependence of the output on the *time* the input is put into the system.

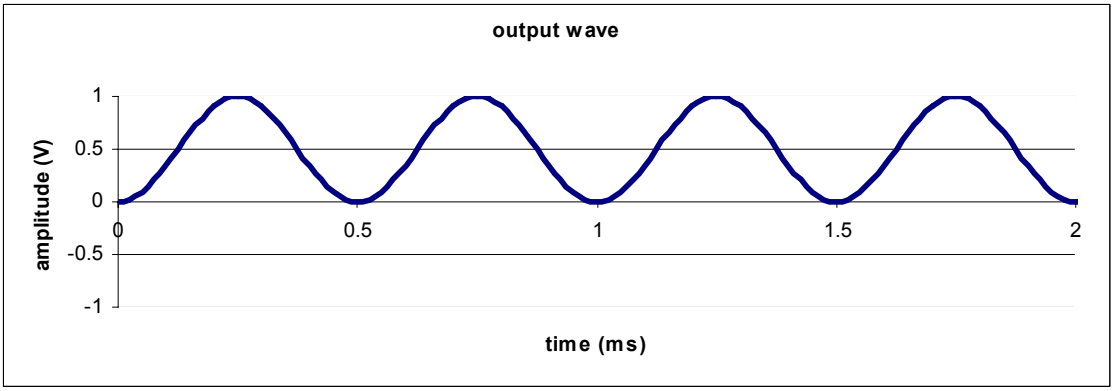
Similar arguments can be readily applied to a system which multiples all input signals by a factor of *m*, as long as *m* does not change over time. If *m* did vary, the system would be linear but time varying.

Chapter 4, exercise 4:

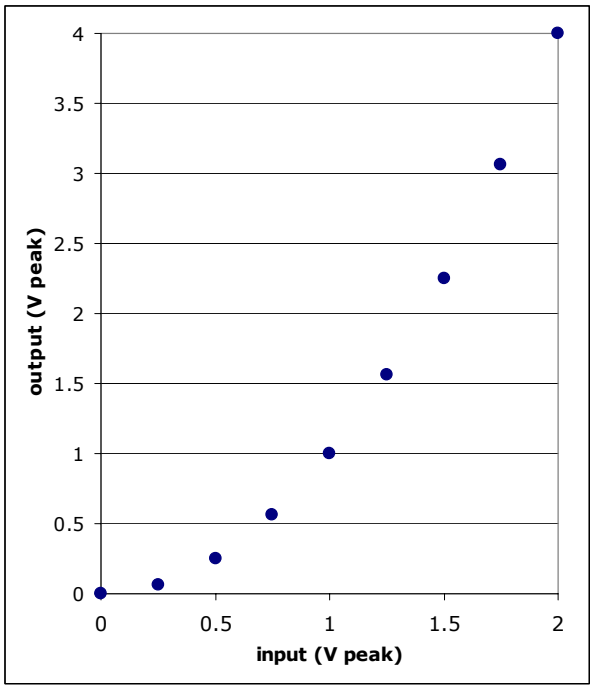
There are at least three main ways to do this.

Perhaps the easiest: sketch the input and output of the system to a sinusoid. Here I have chosen 1 kHz:





Note that the output to a 1 kHz sinusoid is still a sinusoid, but it is at a different frequency. Hence, the system cannot be LTI.



Secondly, plot the input/output function for a sinusoidal input of varying levels. You would then obtain the curve at left. Although this clearly goes through the origin (0,0), it is far from a straight line. Therefore, the system cannot be homogeneous, so cannot be LTI.

Finally, you could demonstrate that this system is not LTI by showing it is not additive, an exercise left to the reader.

Chapter 4, exercise 5:

System ‘D’ has attenuated the input signal, whereas system ‘E’ has both amplified and phase-shifted the input sinusoid. These changes are consistent with a homogeneous system. System ‘Z’ has changed a zero input into a sine wave, so this is not consistent with a homogeneous system, in which a zero input must lead to a zero output. System ‘S’, on the other hand, has made a zero output from an input of some non-zero amplitude, and this is consistent with a homogeneous system. System ‘H’ appears to be a simple amplifier and this too is consistent with homogeneity.

Chapter 4, exercise 6:

System ‘S’: attenuates the signal, so could be LTI.
System ‘T’: amplifies the signal, so could be LTI.
System ‘U’: shifts the phase of the input sinusoid by 180°, so could be LTI.
System ‘V’: changes the shape of the sinusoid, so *cannot be LTI.*
System ‘W’: shifts the phase of the input sinusoid by 90°, so could be LTI.